USING PRECEDENCE CONSTRAINTS TO MODEL THE GEOMETRY OF OPTIMAL MINING ENVELOPES

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Abstract

Precedence constraints are broadly used in mathematical models and algorithms for mine planning to model, for example, slope angles in open pit operations or connectivity in underground mines. Unfortunately, these constraints do not capture relevant geometrical aspects for the design of the mine like minimum bottom space of pits or smoothness of the economic envelope in block caves.
In this work, we present different approaches that utilize precedence constraints generating economic envelops that are more suitable for design purposes. The numerical experiences show that the proposed techniques produce high value envelops with better geometrical properties and that the methods proposed can be computed efficiently.

INTRODUCTION

Mine planning and design is supported by optimization methods and models that assist the planners to make the best decisions. However, as any approach based on mathematical models, these methods do not consider all the geometrical constraints and therefore many researchers have investigated ways to extend current models and algorithms so that the result from the optimization process is closer to a final design.

In open pit mining, the canonical approach is to use the ultimate pit problem (Lerchs & Grossmann, 1965). This model takes as input a set of blocks B with economic values $v_i \in \mathbb{R}, i \in B$ and a set of precedence arcs $P \subset B \times B$ such that $(i, j) \in P$ means that j is a predecessor of i and must be extracted before j to ensure the stability of the pit walls. Therefore, the ultimate pit problem does not incorporate aspects like the minimum bottom pit space that is necessary for the operation of large loading equipment or a volume (as a set of blocks) that can be easily transformed into an operational design. Indeed, it has been reported that the ultimate pit may provide a poor guide for design when compared to the output of models that include these geometrical constraints (Morales et al., 2022).

In block and panel caving, determining the economic envelope of the mine requires computing the best location of the production level (bottom of the mine) plus the set of mineral columns that constitute the envelope, namely the footprint. Contrarily to the case of the ultimate pit, the de facto approach to determine this economic envelope is purely algorithmic. The user may predefine a certain region to limit the columns to be selected and then, for each possible production level, the algorithm determines the set of columns with positive value and constructs the envelope by maximizing the economic value of each column independently (Diering et al., 2010). As a result, the envelope may contain columns that are next to each other but have very different heights, and the shape of the footprint needs to be adapted to be realistic.

In this work, we propose to address some problems related to the geometry of economic envelopes in open pit mining and block and panel caving mining. Our aim is to present efficient methods, i.e., so that the computational time is not an issue. Specifically,

- For open pit, we address the potential issue of *connectivity*, i.e., ensuring that there are not isolated blocks in the pit. For this, we propose to use the exact same model that is currently used for open pit, but to extend the set P with additional precedences; thus, current efficient algorithms can be used without the need to investigate new algorithmic venues.
- For block and panel caving, we develop a methodology that, for a given elevation, can calculate an optimal economic envelope but such that it is connected and has smooth borders and column heights, i.e., we eliminate the need of pre and post preprocessing of the solutions thus ensuring the optimality of the envelope. To do this, we model the envelope as an inverted open pit and add different types of precedences to ensure the properties mentioned before. Therefore, the approach not only ensures the optimality of the solution, but it is very efficient as fast algorithms like pseudoflow (Hochbaum, 2008; Hochbaum & Chen, 2000) can be used for the computations.
- Finally, also for open pit, we address the problem of minimum bottom space. In this case, we use a simple extension of the ultimate pit problem in which we consider a second set of precedences that can be violated at cost in the objective function and use this to penalize narrow pit bottoms.

We test the three methods mentioned before and show that they can be applied efficiently to obtain more practicable economic envelopes.

RELATED WORK

Open pit design optimization

As mentioned before, the ultimate pit considers precedence constraints to model global slope angles only; however, the design of the pit walls is a much more complex task which must consider several design parameters (Hustrulid et al., 2013; Read & Stacey, 2009).

In their seminal work, (Lerchs & Grossmann, 1965) considered a fixed set of slope predecessors for the blocks thus, Khalokakaie et al. (2000) extended their method to manage multiple slope definitions across the mine by using one angle per cardinal direction interpolating linearly in between. In turn, this approach was later extended to consider better interpolations (Gilani & Sattarvand, 2015; Shishvan & Benndorf, 2016).

In terms of constraints related to the operational space needed by large machinery, several authors have worked on different models, notoriously mathematical programming and specialized algorithms, to address the problem of computing one or several nested pits that comply with geometric constraints. (Bai et al., 2018) uses mathematical programming combined with the application of some simple algorithms to improve the geometry. Tabesh et al., 2014 also use mathematical programming, in their case to define "mining polygons". The resulting formulation is difficult to solve thus they implement a greedy heuristic and postprocessing to improve the geometry. Finally, Nancel-Penard & Morales, 2021 present a mathematical model that finds pit with several properties (connectivity, minimum space, no isthmus) without the need of post processing of the solution. The resulting model is difficult to solve so they apply a preprocessing technique to speed up the computation times.

Block and panel caving footprint optimization

In the case of block and panel caving mines, the resulting footprint must comply with several geotechnical and geometrical considerations. First, the footprint must exclude portions that do not comply with a minimum critical hydraulic radius that facilitates a steady caving process. This radius is commonly estimated through Laubscher's stability graph and depends on the adjusted rock mass rating of the orebody (D. Laubscher, 1994; D. H. Laubscher, 1993). Second, to help the even draw of material during production, the height of draw for close columns must be regular (Diering, 2004; Diering et al., 2010). The uniform draw is desirable to promote the interaction of drawpoints and minimize dilution (Castro & Paredes, 2014), and to prevent operational issues like stress and hang-ups (Brown, 2007; Nezhadshahmohammad et al., 2019).

Despite all the geometrical aspects that should be considered for determining the caving footprint, obtaining a favourable caving shape and limit the effects of the abutment stress in the production level, the current practice in block caving optimization is to manually smooth the footprint outline. This process is usually made by the mining engineer and has received very little attention in the literature. Some examples of this procedure can be found in Gantumur et al., 2016 and Noriega et al., 2018.

As mentioned before, in this work we use precedence constraints to model the connectivity and smoothness of the caving envelope. Indeed, (Elkington et al., 2012) first proposed to use slope constraints to model the regularity of height of draws and (Julio et al., 2015; Vargas et al., 2014) used the same idea, in the context of transitioning from open pit to underground mining and determining caving envelopes under geological uncertainty. However, in this work we use more elaborate precedence constraints to also control minimum height of draw, connectivity of the caving and smoothness of the plant limits.

BLOCK AND PANEL CAVING SMOOTH ECONOMIC ENVELOPE

This section presents a methodology to obtain an optimized economic envelope with smooth geometry for block or panel caving. The geometric body that must generate this set of columns has to consider several geomechanical criteria to achieve an operationally valid design. The first criterion is to comply with a critical hydraulic radius that facilitates constant undercutting. Also, the footprint contour must have a smoothed shape to avoid the concentration of high induced stresses. Mainly, these stresses can generate loss of reserves, punctual collapse or increase in the development cost. Another geometric criterion is the difference between the heights of adjacent columns. This difference heights should not be pronounced. This property facilitates uniform draw which helps to avoid early dilution entry. These shape criteria promote a smoothed geometric body in a horizontal and vertical plane. The traditional approach to defining an envelope computes an envelope with maximum economic value that requires design post-processing to be operationally valid. This is why, the proposed methodology includes these shape constraints in the economic envelope optimization stage. For this purpose, the traditional ultimate pit solver is adapted by inverting the cone and adding a set of precedence arcs.

Currently, the accepted method to find the elevation and economic envelope of block and panel caving mines evaluates each elevation separately and determines the best set of economic columns, thus reporting economic value, grades and tonnages for each level and leaving the decision of the level to the planner. This helps to fractionate the block model, bounded by the optimal floor and the maximum height of a mining column, H_{max}. We propose to use the same general procedure but replace the computation of the economic envelope at each level by an inverted pit with additional precedence constraints to control the shape of the envelope. The advantage of doing this is a better control of the geometry and to take advantage of the fast algorithms that exist to compute pits.

To model the shape of the pit, we define different precedence arcs $i \rightarrow j$, where i and j are blocks of the block model $\mathcal{B}$. The first set of precedences are the vertical arcs, present within a draw column. The arc $i \rightarrow j$ is generated if block j is located immediately below block i, except if block i is located at the base of the envelope (Fig 1. Left). Additionally, to respect the minimum draw height, H_{min}, that a draw column must possess, an arc $i \rightarrow j$ is incorporated into each column, where block i is located at the base of the column and block j is located at a vertical distance equivalent to the minimum draw height. The next set of precedences guarantees smoothness in the vertical and contains of diagonal arcs. The diagonal arcs allow the interaction between adjacent columns to control the height difference. If the height difference between columns must be δ, then arc $i \rightarrow j$ is defined if block j is in a contiguous column and at a vertical distance δ from i (Fig. 1, left). Finally, the third set of precedences contains horizontal arcs, which guarantee a smoothed shape for the footprint. For this, c, i and j are blocks belonging to $\mathcal{B}$, where c is the center of a disk $D(c, R)$, where R is the Euclidean distance between blocks c and i, denoted as $d(c, i)$. Furthermore, block i is the center of the disk $D(i, r)$, with r constant. Then, arc $i \rightarrow j$ is defined only if $d(c, j) \leq R$ and

$d(i,j) \leq r$. As shown in Figure 1 (right), all those blocks that lie within the area of intersection between the disks $D(c,R)$ and $D(i,r)$ will be precedence of block i.

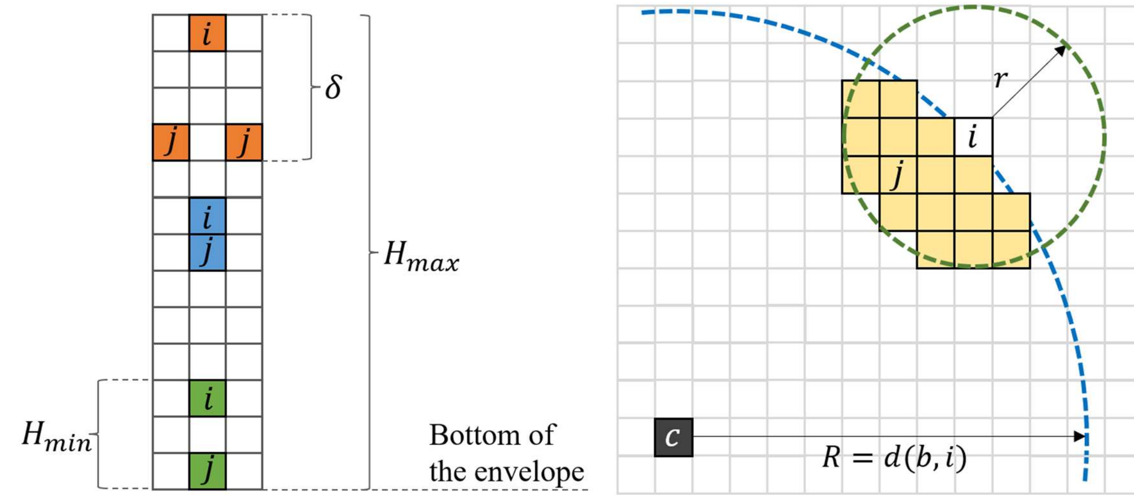
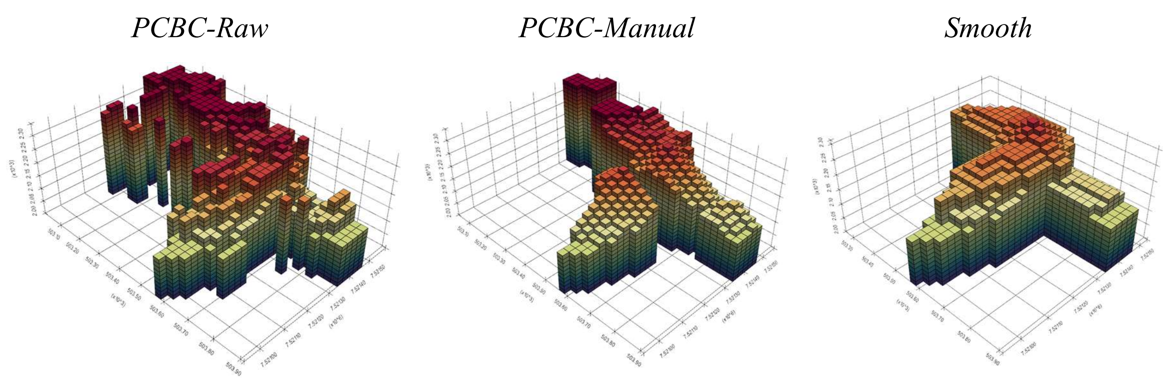


Figure 1. Left: Vertical precedences in green and blue, and diagonal precedences in orange. Right: Horizontal precedences of $\boldsymbol{i}$ in yellow, with *initial point* in $\boldsymbol{c}$ (black).

It should be noted that $\boldsymbol{c}$ is called *initial point*, and one or more can be arbitrarily defined. The shape parameters $\boldsymbol{\delta}$ and $\boldsymbol{r}$ are arbitrarily defined, influencing the smoothness of the economic envelope. For example, the smaller the $\boldsymbol{\delta}$ and the larger the $\boldsymbol{r}$, the smoother the envelope. In our case, a single *initial point* is defined in the higher-grade zone, while $\boldsymbol{\delta}$ and $\boldsymbol{r}$ are calibrated, such that the resulting envelope has acceptable contours. This calibration step, prior to obtaining an acceptable envelope, consists of a simple sensitivity analysis to observe the shape and economic value of the envelopes.

An example of application is performed with the *Delphos3* block model. The PCBC method is compared with the proposed method. First, using the PCBC method, an envelope is computed for each level of the block model, to find the level that delivers an envelope with the highest economic value. This envelope is named *PCBC-Raw*, because it has no design intervention (Fig. 2). Based on this envelope, an operational design is proposed by removing or adding draw columns. The resulting envelope is named *PCBC-Manual* (Fig. 2). Finally, an envelope is computed with the proposed methodology, named *Smooth* (Fig. 2).



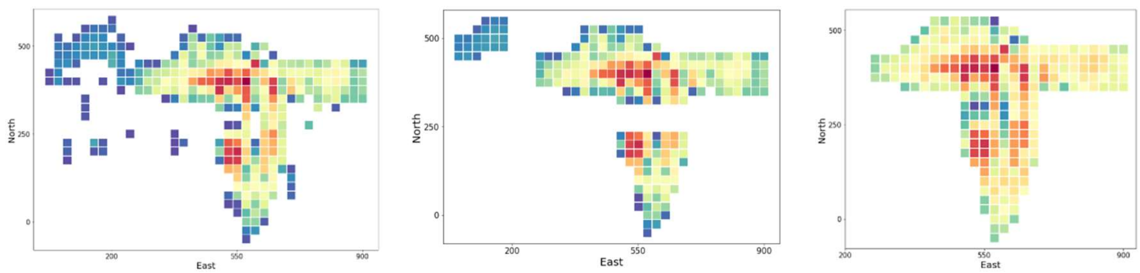


Figure. Shape comparison between *PCBC-Raw*, *PCBC-Manual* and *Smooth*. The top row shows an isometric view (colour scale represents the height of the columns), while the bottom row shows the respective plan view (colour scale represents the cumulative value of each column, with warm colours for high values).

It can be seen in Figure that the shape of the *Smooth* envelope has a similar design to the proposed design of the *PCBC-Manual* envelope. A comparison of the economic values shows the good performance of the proposed methodology (Table 1). The *Smooth* envelope obtains a slightly higher economic value than the *PCBC-Manual* envelope and with good geometrical properties.

Table 1. Value comparison between *PCBC-Raw*, *PCBC-Manual* and *Smooth*.

	Economic Value (MMUSD)	*Total Tonnage (MTons)*	*Waste Tonnage (MTons)*	*Mineral (MTons)*
PCBC-Raw	2,737	128,575	28,302	100,273
PCBC-Manual	2,392	99,347	17,957	81,390
Smooth	2,402	89,676	15,112	74,564

CONNECTED OPEN PIT

This section addresses the generation of a pit that spans different ore bodies relatively close together (Fig. 3, left) and in turn has a design closer to an operational one. The traditional ultimate pit solver tends to generate walls with steep convex sections at the intersection between cones (Fig. 3, right). These convex sections are geomechanically unstable, requiring smoothing at a later design step. To address this problem, the same approach applied in the previous section will be used. We propose to add a second set of precedence arcs to the traditional ultimate pit solver.

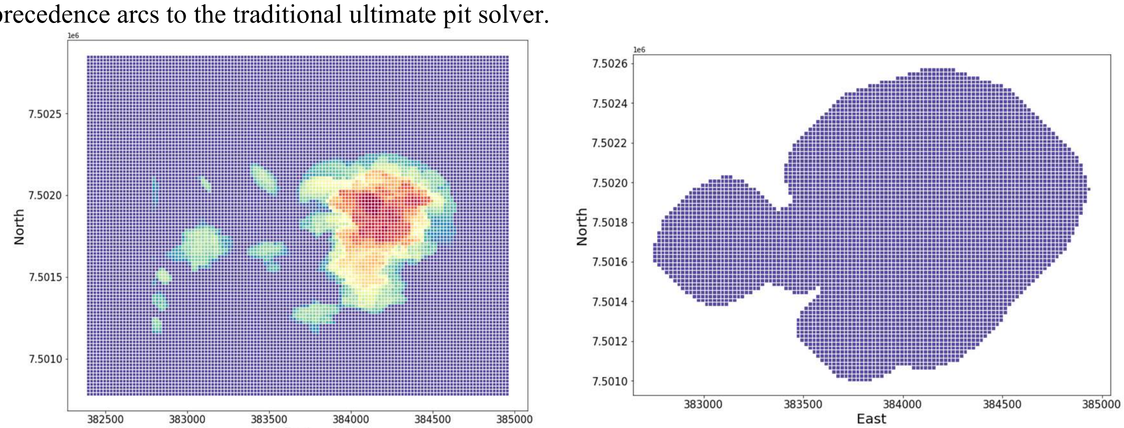


Figure 3. Left: Plan view of block model with different ore bodies. Right: Plan view of standard ultimate pit.

Next, we will explain the methodology and basic notation to define the new set of horizontal precedence arcs. Let c, i and j be blocks belonging to the blocks model $\mathcal{B}$, where c is the center of the ellipse $E(c, a, b)$. Block i is the center of the disk $D(i, r)$ with r constant. The semi-axis a is the Euclidean distance between blocks c and i denoted as $d(c, i)$, while the semi-axis b is constant. Point F_1 is a focus of the ellipse, as is F_2, such that $d(c, F_1) = d(c, F_2) = \sqrt{a^2 - b^2}$. Finally, P any point on the ellipse, only if $d(P, F_1) + d(P, F_2) = 2a$. Is define the arc $i \rightarrow j$ only if $d(j, F_1) + d(j, F_2) \leq 2a$ and $d(i, j) \leq r$. As shown in Figure 4, any block that lies within the area of intersection between the ellipse $E(c, a, b)$ and the disk $D(i, r)$ will be precedence of block i.

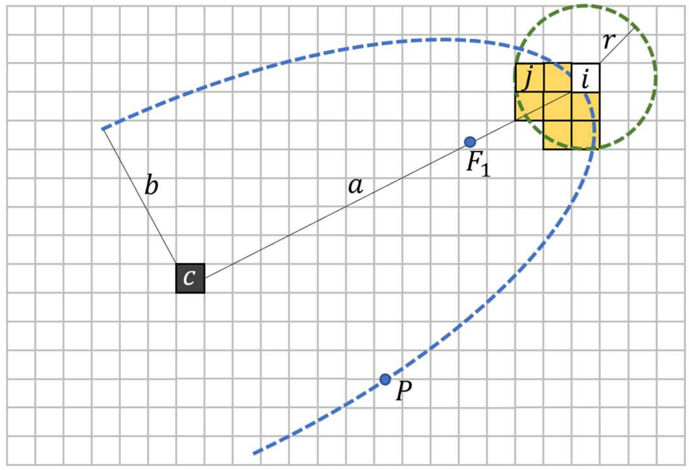


Figure 4. Horizontal precedences of i in yellow, with *initial point* in c (black).

Also, as mentioned above, there may be a set of arbitrarily located *initial points*, c, which are arbitrarily located. Shape parameters r and b with high value will generate less convex sections in the wellbore contour. Our numerical experience is performed with the *g6* block model, where several *initial points* were tested. It is decided to locate five *initial points* (Fig. 5, left), based on the traditional ultimate pit, *UPIT*, specifically on the ore bodies left inside this wellbore. While the shape parameters r and b are calibrated to obtain a well with acceptable shape. The results show that the proposed methodology generates a well (*UPIT-Connected*) with an economic value with a difference of less than 2% (Table 2) and a contour much closer to an operational design (Fig. 5, right). Figure 6 compares both ultimate pits, appreciating in more detail the effect of the new set of horizontal precedences on the convex sections of the slope. The optimized design obtained with our methodology is a better basis for defining a final operational design.

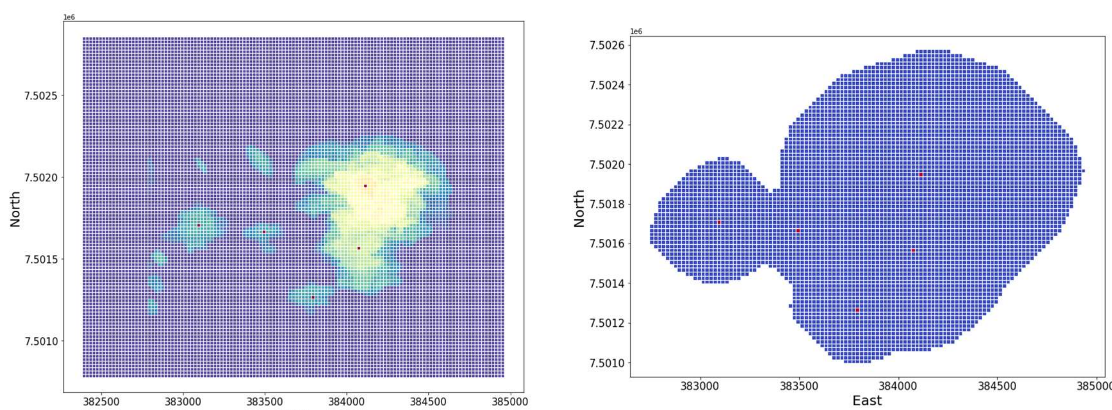


Figure 5. Left: Plan view of block model with five red *initial points*. Right: Plan view of *Connected* pit with five red *initial points*.

Table 2. Value comparison between *UPIT* and *UPIT-Connected*.

	Economic Value (MMUSD)	***Total Tonnage (MTons)***	***Waste Tonnage (MTons)***	***Mineral (MTons)***
UPIT	8,079	482	359	123
UPIT-Connected	7,942	491	368	123

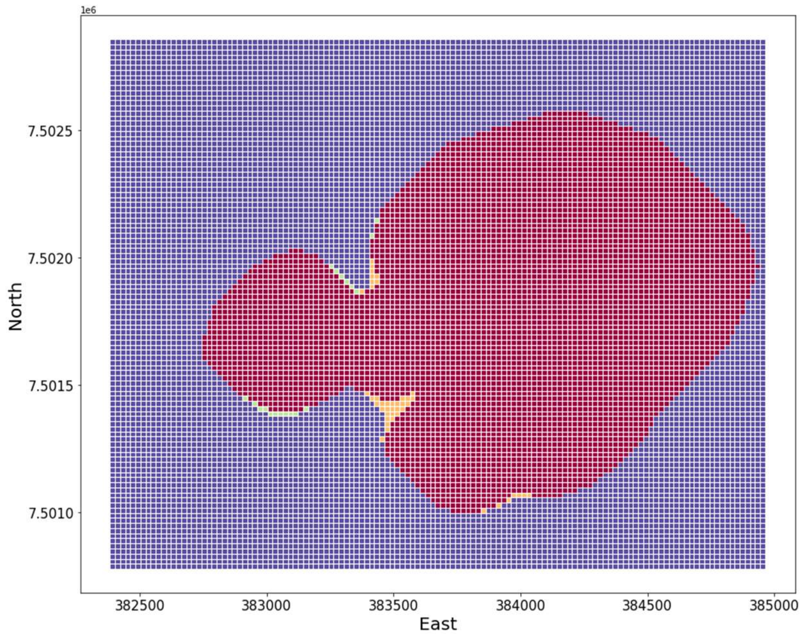


Figure 6. Overlap between *UPIT* and *UPIT-Connected* (Green: *UPIT*, Orange: *UPIT-Connected*, and Red: *UPIT* + *UPIT-Connected*).

OPEN PIT WITH MINIMUM BOTTOM WIDTH

In this section we focus on the problem of defining a set of nested pits with a minimum bottom width. Several works have proposed models to obtain pits with operational space, but they usually result in complex mixed-integer programs that are hard to solve. To address this challenge, we propose a different approach: an extension of the final pit problem to include penalties for not meeting the minimum mining width criteria. The idea behind these penalties is to include a different operational criterion. The minimum width is associated with the operational space required by the loading equipment of the mine. However, smaller widths can be mined if different, smaller equipment is used. The downside of this decision is that smaller equipment is, comparatively, more expensive to operate per ton of moved material. Therefore, a model could allow for smaller mining widths if the profit is enough to pay for the higher mining cost.

To formalize the problem, let us call B the block model and i a particular block and v_i the net value associated with the extraction of block i. P represents the set of precedence constraints. If pair $(i,j) \in P$, then block i can be extracted only if block j is extracted as well. Binary decision variable $x_i, i \in B$ controls the extraction of block i. We introduce a second set of predecessors $Q \subset B \times B$. If $(i,j) \in Q$ we say that j is a *soft predecessor* of i because j is not required to be extracted before i, i.e., the "precedence constraint" in this case can be violated but at a cost $c(i,j)$.

The integer program that models our problem is shown next:

$$
\begin{aligned}
(UPIT^{+}) \quad max \quad & \sum_{i \in B} p_i x_i - \sum_{(i,j) \in Q} c_{ij} y_{ij} \\
& x_i \leq x_j && \forall (i,j) \in P \\
& x_i - x_j \leq y_{ij} && \forall (i,j) \in Q \\
& x_i \in \{0,1\} && i \in B \\
& y_{ij} \in \{0,1\} && \forall (i,j) \in Q
\end{aligned}
$$

As the focus of this paper is the application, we do not provide a mathematical proof of this, however it can be shown that the binary constraints can be relaxed and replaced by $x_i, y_{ij} \in [0,1]$. That is, the problem is indeed a continuous linear program which can be solved in polynomial time.

An important aspect of the implementation is to define where we should apply the soft precedence constraints, or, equivalently, where the bottom of a pit is located. To find a pit's bottom, we first solve the final pit problem without soft precedence arcs. Then, we identify the benches at the bottom of the pit. Finally, we apply the soft precedence arcs for a neighborhood of those benches and solve the resulting model. We explored different penalty cost and minimum widths for our case study. Fig. 7 shows the results of these alternatives for three revenue factors (0.6,0.8 and 1.0) and their comparison with the traditional nested pits approach for the Marvin dataset (53,271 blocks).

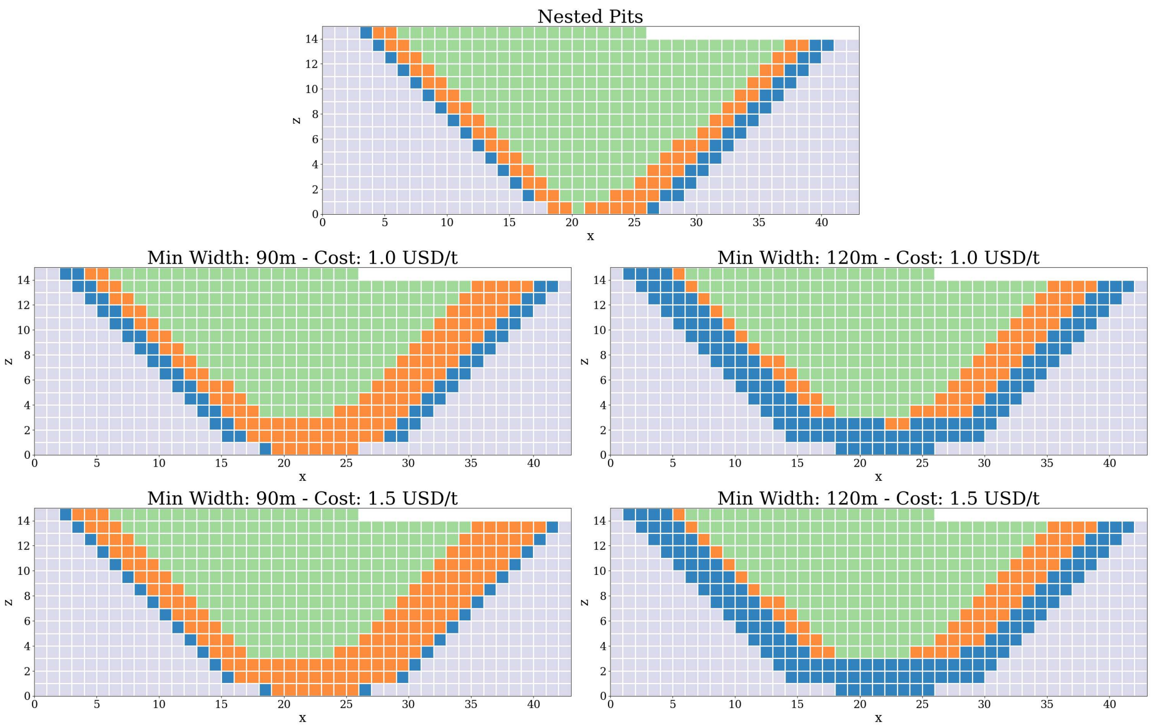


Figure 7. Pits with penalized bottoms and comparison with traditional nested pits. Colors indicate the resulting pit for each revenue factor

As seen in Fig. 7, our model successfully impose a minimum width at the bottom of the pits for each revenue factor. Note that, depending on the case, the pit with minimum bottom constraints might be larger or smaller than the traditional final pit. For instance, for RF=0.6, the pits with soft constraints tend to be smaller than the pits without them. On the contrary, for RF=0.8 and RF=1, the soft constraints generate a larger pit to accommodate for the minimum width requirement. It is also important to notice that, in some cases, the model chooses to not respect the optional minimum width constraints (see, for instance, RF=0.8, Min Width=120m, Cost=1.0 USD/t). For this case, it is more profitable to extract the bottom's blocks with smaller equipment than either not to extract them or to generate a larger bottom.

To further analyze the results, Table 3 shows a comparison between profit and tonnage of our model (UPIT+) and the traditional final pit (UPIT) for the case study presented previously. In terms of value, the inclusion of soft constraints decreases the profit of the resulting pit compared to the traditional final pit. However, the difference is small, ranging from 3% to 9% depending on the case. If we analyze the result of UPIT and apply the penalties for not fulfilling the minimum width requirement, the decrease in value is larger, ranging from 4% and 35% depending on the case. This result shows that our model is able to respect the minimum width constraints without sacrificing a large share of the original profit. At the same time, the comparison between tonnage also demonstrates that the application of UPIT+ could result in a larger or smaller pit compared to UPIT. Finally, computation time is adequate for long-term planning. While our model is more complex than UPIT, its mathematical properties allow for low runtimes.

Table 3. Numerical comparison between UPIT+ and UPIT

			Profit (MMUSD)			*Tonnage (MMt)*		
Min. Width (m)	***Cost (USD/t)***	***RF***	***UPIT+ Penalized***	***UPIT***	***UPIT Penalized***	***UPIT+***	***UPIT***	***UPIT+ Runtime (s)***
90	1	0.6	637.50	669.01	572.1	348.52	376.19	16.87
90	1	0.8	1902.46	1989.91	1849.16	541.38	494.87	26.09
90	1	1	3432.57	3537.98	3382.75	622.11	564.28	24.19
90	1.5	0.6	635.67	669.01	523.65	344.72	376.19	17.68
90	1.5	0.8	1883.17	1989.91	1778.78	555.15	494.87	27.12
90	1.5	1	3395.13	3532.82	3296.93	646.17	571.13	24.71
120	1	0.6	628.28	669.01	432.31	342.04	376.19	17.36
120	1	0.8	1818.45	1989.91	1634.76	446.7	494.87	32.16
120	1	1	3322.25	3538.8	3138.73	662.85	562.33	28.32
120	1.5	0.6	627.08	669.01	313.95	340.44	376.19	21.46
120	1.5	0.8	1811.50	1989.91	1457.19	432.82	494.87	26.45
120	1.5	1	3280.53	3538.8	2938.69	684.86	562.33	30.48

CONCLUSIONS

Conventional methods for optimizing the economic envelope of open pit and block and panel caving mines do not incorporate all the geometrical constraints necessary to generate an operational envelope. Therefore, significant parts of the process of mine design rely strongly on the criteria and skills of the engineers to find a good balance between the envelopes reported by optimization models and feasible geometries for design purposes. Because of this, it is not possible to ensure optimality or evaluate multiple options fast enough to ensure the robustness of the decisions. In terms of research, many authors have addressed these issues, however partially and most of the time by means of complex mathematical formulations that require to be solved by heuristics or that use complementary algorithms to treat the outputs of the optimization phases, i.e., cannot ensure optimality and tend to be difficult to implement and slow to be solved.

In this paper, we apply a known model, namely the ultimate pit problem to address some geometrical issues of open pit and block and panel caving mines. The application of this well-known model ensures that even large instances can be solved very quickly by applying efficient algorithms. Moreover, we apply an extension of the ultimate pit problem in which some precedence constraints can be violated at a cost, to address the problem of minimum bottom space at open pit mines.

The application of the methods shows that it is possible to improve the geometry of economic envelopes significantly and that this can be done efficiently: For block and panel caving mines, the method generates smooth envelopes (with regards to differences in heigh and borders) in short computational times. In open pit mining, the addition of “weak precedence” arcs allows improving the geometrical conditions at the bottom of the pit, and extra horizontal precedence arcs to improve the smoothness and geometry of the borders of the pit. The fact that the methods used for computation are efficient (polynomial algorithms) opens the door to continue to extend the models and to address the issue of robustness of the solutions.

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REFERENCES

Bai, X., Marcotte, D., Gamache, M., Gregory, D., & Lapworth, A. (2018). Automatic generation of feasible mining pushbacks for open pit strategic planning. In *Journal of the Southern African Institute of Mining and Metallurgy* (Vol. 118, Issue 5, pp. 515–530). https://doi.org/10.17159/2411-9717/2018/v118n5a8

Brown, E. (2007). Cave initiation by undercutting. In *Block caving geomechanics* (pp. 191–243). The university of Queensland.

Castro, R. L., & Paredes, P. S. (2014). Empirical observations of dilution in panel caving. *Journal of the Southern African Institute of Mining and Metallurgy*, *114*(6), 455–462.

Diering, T. (2004). Computational considerations for production scheduling of block cave mines. *Proceedings of Mass Mining*, 135–140.

Diering, T., Ritcher, O., & Villa, D. (2010). *Block Cave Production Scheduling Using PCBC* [Report].

Elkington, T., Bates, L., & Ritcher, O. (2012). Block Caving Outline Optimisation. *6th International Conference & Exhibition on Mass Mining*.

Gantumur, J., Godard, R., Prince, F., & Pascoe, C. (2016). *Development and Evaluation Processes of Cave Production Schedules for the Resolution Copper Project*.

Gilani, S.-O., & Sattarvand, J. (2015). A new heuristic non-linear approach for modeling the variable slope angles in open pit mine planning algorithms. *Acta Montanistica Slovaca*, *20*, 251–259. https://doi.org/10.3390/ams20040251

Hochbaum, D. S. (2008). The pseudoflow algorithm: A new algorithm for the maximum-flow problem. In *Operations Research* (Vol. 56, Issue 4, pp. 992–1009). https://doi.org/10.1287/opre.1080.0524

Hochbaum, D. S., & Chen, A. (2000). Performance analysis and best implementations of old and new algorithms for the open-pit mining problem. In *Operations Research* (Vol. 48, Issue 6, pp. 894–914). https://doi.org/10.1287/opre.48.6.894.12392

Hustrulid, W. A., Kuchta, M., & Martin, R. K. (2013). *Open Pit Mine Planning and Design, Two Volume Set & CD-ROM Pack (Book 1—Fundamentals)*. CRC Press. https://doi.org/10.1201/b15068

Julio, C., Amaya, J., Lamghari, A., & Morales, N. (2015). Optimal Economic Envelope of Joint Open-Pit and Underground Mines. In *Proceedings of APCOM* (pp. 343–351).

Khalokakaie, R., Dowd, P. A., & Fowell, R. J. (2000). Lerchs–Grossmann algorithm with variable slope angles. In *Mining Technology* (Vol. 109, Issue 2, pp. 77–85). https://doi.org/10.1179/mnt.2000.109.2.77

Laubscher, D. (1994). Cave Mining: The State of the Art. *Joumal of The Southern African Institute of Mining and Metallurgy*, 279–293.

Laubscher, D. H. (1993). Planning Mass Mining Operations. In *Analysis and Design Methods* (pp. 547–583). Elsevier. https://doi.org/10.1016/B978-0-08-040615-2.50028-9

Lerchs, H., & Grossmann, I. F. (1965). *Optimum Design of Open-Pit Mines: Vol. LXVIII* (pp. 17–24).

Morales, N., Nancel-Penard, P., & Espejo, N. (2022). Development and analysis of a methodology to generate operational open-pit mine ramp designs automatically. In *Optimization and Engineering*.

Nancel-Penard, P., & Morales, N. (2021). Optimizing pushback design considering minimum mining width for open pit strategic planning. In *Engineering Optimization*.

Nezhadshahmohammad, F., Aghababaei, H., & Pourrahimian, Y. (2019). Conditional draw control system in block-cave production scheduling using mathematical programming. *International Journal of Mining, Reclamation and Environment*, *33*(4), 223–246. https://doi.org/10.1080/17480930.2017.1385155

Noriega, R., Pourrahimian, Y., Victor, W. L., Potvin, Y., & Jakubec, J. (2018). *Optimisation of the undercut level elevation in block caving mines using a mathematical programming framework* (pp. 363–372). Australian Centre for Geomechanics. https://doi.org/10.36487/ACG_rep/1815_25_Noriega

Read, J., & Stacey, P. (2009). *Guidelines for Open Pit Slope Design*. CRC Press. https://doi.org/10.1071/9780643101104

Shishvan, M. S., & Benndorf, J. (2016). The effect of geological uncertainty on achieving short-term targets: A quantitative approach using stochastic process simulation. In *Journal of the Southern African Institute of Mining and Metallurgy* (Vol. 116, Issue 3, pp. 259–264). https://doi.org/10.17159/2411-9717/2016/v116n3a7

Tabesh, M., Mieth, C., & Askari-Nasab, H. (2014). A multi-step approach to long-term open-pit production planning. In *International Journal of Mining and Mineral Engineering* (Vol. 5, Issue 4, pp. 273–298). https://doi.org/10.1504/IJMME.2014.066577

Vargas, E., Morales, N., & Emery, X. (2014). Footprint and economic envelope calculation for Block/Panel Caving mines under geological uncertainty. In *Proceedings of Caving* (pp. 449–456).