Multi-echelon multi-product distribution network considering heterogeneous capacitated vehicles

Cristian Aguayo^{*}, Jorge Amaya[†], Sabine Limbourg[‡]

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Abstract

We consider various products to be distributed from a set of firms to customers located in several cities. A freight transport operator has several types of vehicles for collecting or picking up products from different firms and delivering them directly to customers or through depots or satellites. The satellites are intermediate transshipment facilities that have to be optimally located from a set of sites such as parking lots or adequate real estate. This paper addresses the optimal design of a multi-echelon multi-product distribution network considering heterogeneous capacitated vehicles. The relevance of the problem comes from reducing the congestion and energy transportation savings. The problem is formulated as a mixed-integer linear program, and the validity of the model is tested on medium-small size instances. To solve problems of more realistic size, we propose a matheuristic.

Keywords: Multi-echelon; City Logistics; Multi-product; Heuristics

1 Introduction

Today, more than 50% of the world's population lives in cities; the urban population is projected to reach up to 85% by 2100 [30]. The current and expected growing number of people living and working in cities and the limited space available inside city centres induce an increasing exchange of inbound and outbound freight flows between city centres and their surrounding regions. In their study in the United Kingdom, [4] show that the shared freight transport in a city varies between 15% and 40%, depending on the city size. Moreover, [11] pointed out that freight movement in a city represents between 20% to 30% of the vehicle kilometres, but it is responsible for up to 60% of all CO_2 emissions generated by urban traffics. Studies on Paris showed in 2011 that up to 1.6 million deliveries or collections are made per week in the city [12]. The study of [8] summarised 30 surveys regarding urban freight activities; most of the freight transports are carried out by vans, up to 42% of delivery activity. Of course, these figures vary between cities and strongly depend on local situations.

Urban freight transports provide economic benefits to society but are also responsible for negative externalities such as congestion, air and water pollution, climate change, accidents, and noise

^{*}Department of Industrial Engineering, University of Chile

[†]Center for Mahematical Modeling and Department of Mathematical Engineering, University of Chile

[‡]HEC - Management School, University of Liège

[34]. They are more polluting than long-distance freight transports because of the increased fuel consumption due to the frequent stops on their delivery route. Moreover, due to traffic congestion, most transport vehicles are using alternative routes associated with a longer transport time, higher transport costs, and negative impact on society and the quality of life. In the current Transport White Paper [17], the European Union (EU) presents a roadmap for a more competitive and sustainable European transport system. Concerning Urban Freight Transport, responsible for about a quarter of CO_2 emissions of the transport sector, one of the goals of the EU is to achieve essentially CO_2 -free city logistics in major urban centres by 2030 by developing and deploying new and sustainable fuels and propulsion systems. The gradual phasing out of conventionally fuelled vehicles from the urban environment reduces oil dependence, greenhouse gas emissions and local air and noise pollution.

To meet European air quality standards, authorities of some major European cities have already introduced regulatory measures, and access restrictions are one of the most applied measures to control urban traffics in specific areas of the cities. There are several types of access restrictions, from time windows, emissions and noise limits to vehicle weight and size. Policy measures can be of various types, such as the implementation of Low Emission Zones (LEZs) as described in [3, 15, 33, 27]. In addition, LEZs in some urban areas are limited to freight vehicles that meet specific emissions standards. The growing use of urban trucks based on electric, hydrogen and hybrid technologies or non-motorised transport such as bikes [36, 5] helps reducing not only pollutant emissions but also noise and road congestion by making night deliveries and avoiding morning and afternoon peak periods. Moreover, the use of low-emission fleets also allows mitigating the transportation sector dependence on volatile fuel prices as electric engines may be powered using renewable energy sources such as wind and solar energy. Nevertheless, the concept of LEZs requires a considerable workload and coordination by the municipalities, especially to control and enforce all emission requirements in the city to minimise the probability of unauthorised access into the city. In [35], an overview of more than 20 projects, city measures or initiatives to organise efficient deliveries inside cities are presented. Among their key findings, the authors found that (1) the use of Urban Consolidation Centers (UCCs) contributes to successful results in terms of energy efficiency, congestion and gas emissions (2) tactical planning is vital for freight carriers using consolidation operations (3) delivery and servicing plans are promising solutions to generate common goods collection and additional services (e.g. removal of waste), and (4) the use of environmentally friendly vehicles combined with low emission zones and UCCs allows to reduce the environmental impact.

Measures related to land use planning and infrastructure are usually very cost-intensive and thus include long time planning and long implementations periods. These measures range from on-street as well as off-street loading zones to specific delivery zones and collecting points. Also, UCCs, inside and outside the city, are part of these measures. Although a UCC provides efficiencies in terms of transport utilisation, it has high set-up costs as well as operating costs [25]. It represents an extra organisational and financial effort for most companies because it is an additional handling stage in their supply chain. Therefore, some UCCs are financed by public funding [3].

Over the last years, the consolidation of goods has been downscaled; this idea is associated with micro-depots. Micro-depots became the centre of attention of many transport services that are dealing with light freight. Especially the consolidation concept is increasingly adopted by CEP (Courier-, Express-, Parcels-)services, which handle shipments up to 30 kg; in different European cities. Typically, CEP-services transport small shipments to private and commercial customers [40]. Their services cover Business-to-Customer (B2C), Business-to-Business (B2B) and also Customerto-Customer (C2C) markets which result in a high amount of shipments per day [40]. Most of these micro-depot approaches deal with the supply by larger transport vehicles which deliver a massive amount of parcels to the micro-depot [21]. Besides this, the concept of micro-depots also combines foot deliveries with the use of bicycles for the so-called *last mile* in the inner-city, or even delivery men can use simple hand trucks for short delivery tours. Therefore, the location of micro-depots can differ by the project (see, for instance, La Petite Reine founded in 2001 in Paris and was spread over many cities after 2010 [12]. The micro-depot locations range from parked mobile vehicles or containers to adequate real estates, which consolidate goods and transfer them to the delivery vehicles. An additional stage of consolidation, achieved with minisatellite platforms, is proposed in [2]. Another extension is based on the location of mini hubs, i.e. sections of curbs that do not require any investment [29].

It has also been underlined that, until recently, most of the available literature on urban freight distribution exists of companies and governmental reports [41], articulating the need for more scientific advice. A review of the literature [10] reveals that general concepts related to city logistics are proposed but that very few models and methods are devoted to their design, planning, management, and evaluation. In their review, [37] have identified future research directions in bundling networks, vehicles fleet and vehicle routing problems. [24] study the impact of location, fleet composition, and routing on emissions in a city logistics context. They show that because of the effect of speed zones, it may be advantageous to follow circuitous routes to achieve faster speeds and hence lower costs and CO_2 emissions.

Multi-echelon distribution was first presented in [32]. The authors considered the particular case of a two-echelon capacitated vehicle routing problem (2E-CVRP) with a single product. This two-echelon schema only allows transportation from the main depot to different satellites and from satellites to customers. Each customer is served by one and only one vehicle. 2 heuristics are proposed, and they were tested in benchmark instances of CVRP from [9]. In their literature revue, [2] state that the offered methods do not deal with the freight movements except as approximations of base volumes on the arcs representing road of the city.

This paper aims to determine how to efficiently distribute various products made by small and medium firms to customers from several cities. Some of these firms have vehicles that can deliver products to the depots of the freight transport operator or *satellites*. The satellites have to be optimally located from a set of facilities such as parking lots or adequate real estate. The freight transport operator has several types of vehicles for collecting or picking up products from different firms, and delivering them directly to customers or through satellites. Our main goal is to propose a mixed-integer linear program for the specific case of a Multi-Echelon Multi-Product distribution network considering Heterogeneous capacitated Vehicles (MEMPHV). The validity of this model is tested on medium-small size instances. To solve problems of a more realistic size, we propose a matheuristic.

The remainder of the paper is organised as follows. In Section 2, we present a formal description of the MEMPHV. In Section 3, we propose a mixed-integer linear formulation, whereas, in Section

4, we introduce a metaheuristic. The computational results and related conclusions are discussed in Section 5, while closing remarks are given in Section 6, along with potential future perspectives.

2 Problem description

Freight transport operator, like third-party logistics provider, brings goods to customers in the urban area. Freight transport operators and shippers want to strengthen the synergy effects and cost savings by consolidating product flow at depots or satellites, increasing the utilisation rate of vehicles, and using route and tour optimisation tools. Their objective is to reduce their costs while meeting the expectations of their customers and following rules and policies regulating activities and traffic.

Firms have various products characterised by a specific volume and weight. As the carrying capacity of every vehicle is limited by space or weight, the weight and volume feature of the products plays a key role in our model. To ensure at least one feasible solution, some conditions are assumed to be satisfied: e.g., the weight of each product is supposed to be less than or equal to the maximum capacity of the vehicle or facility used. According to [22], goods where the load weight is the restricting factor are, for example, coal, ore, oil or some chemical products, whereas products with volume as the limiting factor are vehicle parts, clothes and consumer articles. The cost of moving a product is based on both its weight and volume, by converting the volume into a volumetric weight. The freight transport operator charges per kilogram for the maximum between the actual weight or the volumetric weight using a volume-to-weight conversion factor. For instance, the International Air Transport Association (IATA) recognises a volume-to-weight conversion factor of 6 m³ to 1 ton in air transport, while express freight-courier mainly consider 5 m³ to 1 ton [13].

We assume that products are loaded into standardised containers to combine them into larger units easily. We also assume that all demanded products must be delivered and that each customer is served by one and only one vehicle. There are two kinds of intermediate locations for the product flows from firms to customers. They play two roles that are the consolidation of flows and transshipment between vehicles. The two types of considered facilities are depots and satellites with more limited warehousing capacity. The depots' locations are fixed, and the satellites have to be located within the urban zone from potential locations such as parking lots adequate real estate. The satellites can be located near an active zone with a road network mainly characterised by alleys, narrow roadways such as the historic city centre.

Figure 1 represents the flows of the distribution. Some firms can deliver products to depots (red arc) or satellites (purple arcs). If a firm does not have any vehicles for transporting their products, vehicles from the freight transport operator perform a tour pick them up (blue arcs); that is the first echelon of the distribution network. The freight transport operator can consolidate the products at the depots or satellites. He/she can bring them from the depot to a satellite close to the city centre; that is the second echelon of the distribution network. The exchange between depots is not considered. Products are loaded to vehicles appropriate for the city centre, delivered to customers (green and black arcs), which is the third echelon of the network. Each vehicle, depot, and satellite have a maximum capacity in terms of volume and weight.

The freight transport operator has a heterogeneous fleet of vehicles to cope with the utilisation of

environment-friendly vehicles and the traffic rules. The vehicles (trucks, vans, electric vehicles, cargo bikes) are characterised by capacities in terms of volume and weight and costs. Smaller vehicles are only used between satellite and customers, whereas larger vehicles cannot do the last mile. Finally, each vehicle returns empty to its depot. The objective is to find the location of satellites and the routing of the heterogeneous fleet to minimise the delivery cost.



Figure 1: Representation of the distribution network.

3 Mathematical formulation

The MEMPHV problem is defined on a directed graph $G = (\mathcal{N}, \mathcal{A})$ where \mathcal{N} is the set of nodes indexed by $i, j \in \{0, ..., n\}$ and $\mathcal{A} = \{(i, j) : i, j \in \mathcal{N}, i \neq j\}$ is the arc set. \mathcal{N} is composed of the sets of firms \mathcal{F} , depots \mathcal{D} , satellites \mathcal{S} , and customers \mathcal{C} . In other words, $\mathcal{N} = \mathcal{F} \cup \mathcal{D} \cup \mathcal{S} \cup \mathcal{C}$. Let P_f denote the set of products from firm f $(f \in \mathcal{F}), \mathcal{P} = \bigcup_{f \in \mathcal{F}} P_f$ denote the set of all products, and V_i be the set of available vehicles at i $(i \in \mathcal{D} \cup \mathcal{S})$.

For notation purposes, the set of firms without vehicles is denoted by $F = \{f \in \mathcal{F} : V_f = \emptyset\}$, the set of firms with vehicles by $\overline{F} = \mathcal{F} \setminus F$, the set of all vehicles from a node set $\Xi \in \{\mathcal{F}, \mathcal{D}, \mathcal{S}\}$ by $V_{\Xi} = \bigcup_{\xi \in \Xi} V_{\xi}$, and the set of all vehicles by $\mathcal{K} = V_{\mathcal{F}} \cup V_{\mathcal{D}} \cup V_{\mathcal{S}}$.

Each customer $c \in C$ has a demand of D_{pc} units of product $p \in \mathcal{P}$. A unit of product p has a volume ν_p , a gross weight ω_p , including the product, packaging and a returnable transport item such as pallet, if any and a chargeable weight ω'_p . The product can be handled at a facility $i \in \mathcal{D} \cup S$, a depot or a satellite. The facility has a capacity Λ_i in terms of volume and Ω_i in terms of delivery

weight. The operating cost associated with the use of satellite $s \in S$ is ε_s .

A vehicle $k \in \mathcal{K}$ with a volume capacity Φ_k and a weight capacity Θ_k can be used to move the product between *i* and $j \in \{0, ..., n\}$, r_{ij} km away at a cost divided into:

 ρ_k : a cost per distance unit; γ_k : a cost factor per chargeable weight unit and per distance unit; δ_k : a fixed cost to use the vehicle k.

where $\rho_k, \gamma_k, \delta_k > 0$.

Some vehicles could not be allowed to go to certain nodes. For example, vehicles from depots cannot visit other depots, or vehicles with high pollutant emissions cannot visit nodes in a LEZ. For all of those cases, the parameter α_{ik} takes the value 1 if vehicle k can visit node i, and 0 otherwise. Due to the fact that a vehicle can transport multiple products, the following decision variables have to be integer:

q_{pijk} units of p passing (i, j) in vehicle k	$\forall p \in \mathcal{P}, \forall i, j \in \mathcal{N}, \forall k \in \mathcal{K}$
m_{pik} units of p to be transported to i in vehicle k	$\forall p \in \mathcal{P}, i \in \mathcal{N} \setminus \mathcal{F}, \forall k \in \mathcal{K}$
x_{pfk} units of p picked from f by vehicle k	$\forall p \in \mathcal{P}, \forall f \in \mathcal{F}, \forall k \in \mathcal{K}$

whereas the decision variables related to the use of vehicles, arcs and satellites have to be binary:

$$y_{k} = \begin{cases} 1 & \text{if the vehicle } k \text{ is used} \\ 0 & \text{otherwise} \end{cases} \qquad \forall k \in \mathcal{K} \\ z_{kc} = \begin{cases} 1 & \text{if the vehicle } k \text{ serves customer } c \\ 0 & \text{otherwise} \end{cases} \qquad \forall k \in \mathcal{K}, \forall c \in \mathcal{C} \\ \forall k \in \mathcal{K}, \forall c \in \mathcal{C} \end{cases} \\ w_{ijk} = \begin{cases} 1 & \text{if the arc } (i,j) \in \mathcal{A} \text{ is used by vehicle } k \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, j \in \mathcal{N}, \forall k \in \mathcal{K} \\ u_{s} = \begin{cases} 1 & \text{if the satellite } s \text{ is opened} \\ 0 & \text{otherwise} \end{cases} \qquad \forall s \in \mathcal{S} \end{cases}$$

The formulation of the MEMPHV can be written as follow:

$$\min\sum_{k\in\mathcal{K}\setminus\overline{F}}\delta_k y_k + \sum_{s\in\mathcal{S}}\varepsilon_s u_s + \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}}\sum_{k\in\mathcal{K}\setminus\overline{F}}r_{ij}\rho_k w_{ijk} + \sum_{p\in\mathcal{P}}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}}\sum_{k\in\mathcal{K}\setminus\overline{F}}r_{ij}\omega_p'\gamma_k q_{pijk}$$
(1)

subject to:

$$m_{pck} = D_{pc} z_{kc} \qquad \forall p \in \mathcal{P}, \forall c \in \mathcal{C}, \forall k \in \mathcal{K}$$
(2)
$$\sum_{k \in \mathcal{K}} z_{kc} = 1 \qquad \forall c \in \mathcal{C}$$
(3)

$$\sum_{k \in \mathcal{K}} w_{ick} = z_{kc} \qquad \forall c \in \mathcal{C}, \forall k \in \mathcal{K}$$

$$(4)$$

$$\forall p \in \mathcal{P}, \forall c \in \mathcal{C}, \forall k \in \mathcal{K}$$
(5)

$$\forall p \in \mathcal{P}, \forall s \in \mathcal{S}, \forall k \in V_{\mathcal{F}} \cup V_{\mathcal{D}}$$
(6)

$$\forall p \in \mathcal{P}, \forall d \in \mathcal{D}, \forall k \in V_{\mathcal{F}}$$
(7)

$$\sum_{i \in \mathcal{N}} q_{pifk} - \sum_{l \in \mathcal{N}} q_{pflk} = -x_{pfk} \qquad \forall p \in \mathcal{P}, \forall f \in \mathcal{F}, \forall k \in V_{\mathcal{D}}$$
(8)

$$\sum_{j \in \mathcal{N}} \sum_{k \in V_f} q_{pfjk} + \sum_{k \in V_{\mathcal{D}}} x_{pfk} = \sum_{c \in C} D_{pc} \qquad \forall f \in \mathcal{F}, \forall p \in P_f$$
(9)

 $\sum_{i \in \mathcal{N}} q_{pick} - \sum_{l \in \mathcal{N}} q_{pclk} = D_{pc} z_{kc}$

 $\sum_{i \in \mathcal{N}} q_{pisk} - \sum_{l \in \mathcal{N}} q_{pslk} = m_{psk}$

 $\sum_{i \in \mathcal{N}} q_{pidk} - \sum_{l \in \mathcal{N}} q_{pdlk} = m_{pdk}$

 $\sum_{p \in \mathcal{P}} \sum_{j \in \mathcal{N}} q_{pijk} \ \nu_p \le \Phi_k \ y_k$

 $\sum_{p \in \mathcal{P}} \sum_{j \in \mathcal{N}} q_{pijk} \ \omega_p \le \Theta_k \ y_k$

 $w_{ijk} \le y_k$

$$\sum_{j \in \mathcal{N}} \sum_{k \in V_d} q_{pijk} = \sum_{k \in \mathcal{K}} m_{pik} \qquad \forall p \in \mathcal{P}, \forall i \in \mathcal{D} \cup \mathcal{S}$$
(10)

$$\sum_{k \in \mathcal{K}} \sum_{p \in \mathcal{P}} m_{pdk} \ \nu_p \le \Lambda_d \qquad \qquad \forall d \in \mathcal{D}$$
(11)

$$\sum_{k \in \mathcal{K}} \sum_{p \in \mathcal{P}} m_{pdk} \ \omega_p \le \Omega_d \qquad \qquad \forall d \in \mathcal{D}$$
(12)

$$\sum_{k \in \mathcal{K}} \sum_{p \in \mathcal{P}} m_{psk} \ \nu_p \leq \Lambda_s \ u_s \qquad \forall s \in \mathcal{S}$$

$$\sum_{k \in \mathcal{K}} \sum_{p \in \mathcal{P}} m_{psk} \ \omega_p \leq \Omega_s \ u_s \qquad \forall s \in \mathcal{S}$$
(13)

$$\forall s \in \mathcal{S} \tag{14}$$

$$\forall i \in \mathcal{N}, \forall k \in \mathcal{K} \tag{15}$$

$$\forall i \in \mathcal{N}, \forall k \in \mathcal{K} \tag{16}$$

$$\forall i, j \in \mathcal{N}, \forall k \in \mathcal{K} \tag{17}$$

$$\begin{array}{ll}
q_{pijk} \ \nu_p \leq \Phi_k \ w_{ijk} & \forall p \in \mathcal{P}, \forall i, j \in \mathcal{N}, \forall k \in \mathcal{K} \\
q_{pijk} \ \omega_p \leq \Theta_k \ w_{ijk} & \forall p \in \mathcal{P}, \forall i, j \in \mathcal{N}, \forall k \in \mathcal{K} \\
\end{array} \tag{18}$$

$$\forall i \in \mathcal{N}, \forall k \in \mathcal{K}$$

$$(20)$$

$$\sum_{i \in \mathcal{N}} w_{ijk} = \sum_{l \in \mathcal{N}} w_{jlk} \qquad \forall j \in \mathcal{N}, \forall k \in \mathcal{K} \qquad (20)$$
$$\sum_{j \in \mathcal{N}} \sum_{k \in V_i} w_{ijk} \le |V_i| \qquad \forall i \in \mathcal{N} \setminus \mathcal{C} \qquad (21)$$

$$\sum_{i\in\mathcal{N}}^{k\in\mathcal{V}_i} w_{ijk} \le \alpha_{jk} \qquad \forall j\in\mathcal{N}, \forall k\in\mathcal{K}$$
(22)

 $x_{pfk} = 0$ $\forall p \in \mathcal{P}, \forall f \in \mathcal{F} \setminus F, \forall k \in \mathcal{K}$ (23)

$$q_{pifk} = 0 \quad \gamma \quad \forall p \in \mathcal{P}, \forall i \in \mathcal{N}, \forall f \in \mathcal{F}, \forall k \in V_f$$
(24)

The objective function (1) minimises the total cost. The first sum of the objective function corresponds to the costs of the used vehicles; the second sum corresponds to the costs to locate satellites; the third sum is the cost to travel the distance; and the last term represents the extra cost to transport heavy products. Constraints (2) state that the demands have to be satisfied, and constraints 3 that each customer is served by one and only one vehicle. Constraints (4) ensure that each arc to a customer can be used only by the vehicle serving the considered customer. Constraints (5)-(8) are related to flow conservations; at the customers (5), at the satellites (6), at the depot (7), and at the firms (8). Constraints (9) guarantee that the total quantity of each product leaving each firm corresponds to the same delivered quantity, whereas Constraints (10) guarantee that the total quantity of each product leaving each depot and each satellite is transported by vehicles. Constraints (11)-(16) are related to capacity. Constraints (11) and (12) ensure that the depot's capacity in terms of volume and respectively of weight not exceeded. Constraints (13) and (14)ensure that, if the satellite is located, its capacity in terms of volume and respectively of weight is not exceeded. Constraints (15) and (16) ensure that the vehicle's capacity in terms of volume and respectively of weight is not exceeded. Constraints (17) stipulate that if an arc (i, j) is used by a vehicle k, this vehicle is also used. Constraints (18) and (19) define the bounds in terms of volume and weight of each vehicle and each arc. Constraints (20) ensure that the number of vehicles arriving and leaving each node is the same, and constraints (21) that the number of vehicles leaving each firm (respectively each depot and each satellite) do not exceed the number of available vehicles at the firm (respectively depot and satellite). Constraints (22) enforce each vehicle to pass at most once per node only if that vehicle is allowed to pass through that node. For each firm with vehicles, constraints (23) ensure that its products vehicles can only be sent products in its vehicles. Finally, constraints (24) state that vehicles do the pick-up before visiting satellites or customers.

4 Application

In this section, we explain how our model can be applied to a realistic case. Our data represent realworld practices that are seldom available at a granularity level that can allow for accurate research. We show in Table 1 some typical examples of products to be delivered.

Product	$\nu_p \ (10^{-3} \mathrm{m}^3)$	$\omega_p \; (\mathrm{kg})$	ω_p' (kg)
Pizza	6	0.5	1.2
Shoes	10	1.4	2
5 reams of typing paper	20	12.5	12.5
2 pillows	160	2.7	32
Case of beer	45	16	16

Table 1: Product data.

According to [22], the energy consumption of freight transport depends on various factors such as vehicle type, load factor, cargo specification, driving conditions, traffic route or transport distance. Moreover, evaluation of transportation costs depends on parameters such as location, traffic density and traffic jam, vehicle characteristics, meteorological condition, number of stops, the gradient of the road track, and speed. Assessing the economic costs of congestion involves several parameters, and assumptions [26]. Besides, the interaction between road transport emissions and street structures (e.g. the slope on which the vehicle evolves) also plays an essential role [1]. In [26] the authors show that the average operating cost per tonne.km can vary by a factor of more than two from country to country. The obtained costs are compatible with those considered in [23]. To assess the parameter γ_k , to consider that higher chargeable weight increase cost, we assume that the cost related to energy, tires, and maintenance are double for a full load vehicle compared to an empty vehicle. A simple proportional calculation deduces the intermediate values. Table 2 gives an overview of the characteristic assumptions, considering that the labour cost is $15 \in$ per hour. Our assessments are consistent with expectations. According to [16], the average cost of parking near the historic city centre of European cities is $3 \in$ an hour, ranging from 0.5 to 7.65 \in .

Table 2: Vehicle characteristics.

Vehicle	Bike	Van	Truck
Φ (m ³)	1	16	50
Θ (kg)	200	2000	12000
$\delta ~(\in)$	6	30	55
Energy, tires and maintenance (\in/km)	0.05	0.2	0.4
Average speed (km/h)	10	30	50
Labour cost (\in/km)	1.5	0.5	0.3
$\rho \; (\in/\mathrm{km})$	1.55	0.7	0.7
$\gamma ~(\in/(\text{kg.km}))$	0.0005	0.0002	0.00007

We have tested our mathematical model to check the model's validity and get some insights that could help us develop heuristics for large-size instances. The optimisation steps have been run on an Intel Xeon CPU ES-2620, 2.10GHz workstation with 32.0 GB RAM and 64-bit Windows 10 Pro. The code is implemented in Python using the Gurobi 9 library as a Branch-and-Bound solver with default parametrisation.

5 Heuristic strategy

To solve the problem, a matheuristic that decompose the original problem is proposed. The decomposition consists of determining the set of nodes to be visited by each vehicle and then determining the optimal route for every vehicle. To determine the set of nodes for each vehicle, two steps are distinguished. The first step is to assign every customer to one and only one vehicle. After this step, each node in $\mathcal{D} \cup \mathcal{S}$ with at least one vehicle serving customer will have demand to be satisfied. Therefore, the second step is to assign other(s) vehicle(s) to satisfy those new demands. The decision of which satellites are opened, and the decision of which vehicles are used is implicit in these steps.

5.1 Obtaining initial feasible solution

To decide which vehicle serves which node, the cost of a vehicle k serving a node i (SC_{ik}) is estimated. If $k \in V_S$, SC_{ik} also includes the cost of transportation to its respective satellite, as well as the operating cost of its satellite.

To determine the optimal routes, a CVRP problem will be solved for each vehicle and its respective set of nodes. To describe the node-vehicle assignment algorithm, we define the sets:

$$S = \emptyset$$
: opened satellites
 $N_k = \emptyset$: nodes to be visited by vehicle k

Also, we define the following parameters:

Ω_i' :	remaining weight capacity, initial value Ω_i	$\forall i \in \mathcal{D} \cup \mathcal{S}$
$\Lambda_i':$	remaining volume capacity, initial value Λ_i	$orall i \in \mathcal{D} \cup \mathcal{S}$
Θ_k' :	remaining weight capacity, initial value Θ_k	$\forall k \in \mathcal{K}$
Φ_k' :	remaining volume capacity, initial value Φ_k	$\forall k \in \mathcal{K}$
D_{pi}' :	units of p remaining demanded by node , initial value ${\cal D}_{pi}$	$\forall i \in \mathcal{C}$
$\bar{\omega_i} = \sum_{p \in \mathcal{P}} D'_{pi} \omega_p :$	total weight demanded by node	$\forall i \in \mathcal{C}$
$\bar{\nu_i} = \sum_{p \in \mathcal{P}} D'_{pi} \nu_p :$	total volume demanded by node	$orall i \in \mathcal{C}$
$m_{pik} = 0:$	units of p to be transported to i by vehicle k	$\forall i \in \mathcal{D} \cup \mathcal{S}, \forall k \in \mathcal{K}$
n(k):	node to which k belongs	$\forall k \in \mathcal{K}$

The cost to serve a customer c with a vehicle k depends on whether or not k belongs to a depot. If k belongs to a depot, SC_{ck} includes the costs to pick the products from firms F, and after that, to deliver them to c. Otherwise, if k belongs to a satellite, SC_{ck} includes the costs to pick the products from firms F by some vehicle $k' \in V_D$, then to transport them to the satellite, and finally, to c by k. In both cases, the picking-up cost is computed using a greedy algorithm that chooses the minimum picking (and delivering) cost. The last delivery cost is the cost of transporting all the products from n(k) to c.

Considering the parameters mentioned before, as well as the sets and an ordered array SC containing all the feasible ($\alpha_{ck} > 0$) serving costs SC_{ck} increasingly ordered. Algorithm 1 shows the procedure for making the customer-vehicle assignment.

Algorithm 1 Customer-vehicle assignment.

1: while $\max_{c \in \mathcal{C}} \{\sum_{p \in \mathcal{P}} D'_{pc}\} \neq 0$ do if $(SC = \emptyset)$ then 2: infeasible 3: break 4: 5: else Select the first $SC_{ck} \in SC$ 6: if $(\omega'_c \leq \min\{\Omega'_{n(k)}, \Theta'_k\})$ and $(\nu'_c \leq \min\{\Lambda'_{n(k)}, \Phi'_k\})$ then 7:
$$\begin{split} \Omega'_{n(k)} &= \Omega'_{n(k)} - \bar{\omega}_c; \ \Lambda'_{n(k)} = \Lambda'_{n(k)} - \bar{\nu}_c \\ \Theta'_k &= \Theta'_k - \bar{\omega}_c; \ \Phi'_k = \Phi'_k - \bar{\nu}_c \\ \text{for } p \in \mathcal{P} \text{ do} \end{split}$$
8: 9: 10: $D'_{pn(k)} = D'_{pn(k)} + D'_{pc}$ 11: $m_{pck} = m_{pck} + D'_{pc}$ 12: $D_{pc}^{\prime} = 0$ 13:14: end for if $(k \notin K)$ then 15: $K = K \cup \{k\}$ 16:for $S_{c'k} \in SC$ do 17: $S_{c'k} = S_{c',k} - \delta_k$ 18:end for 19:end if 20:if $(n(k) \in S)$ and $(n(k) \notin S)$ then 21: $S = S \cup \{n(k)\}$ 22:for $S_{c'k} \in SC$ do 23:24: $S_{c'k} = S_{c',k} - \varepsilon_{n(k)}$ end for 25:end if 26: $N_k = N_k \cup \{c\}$ 27:end if 28:29:end if $SC = SC \setminus \bigcup_{k' \in \mathcal{K}} \{SC_{ck'}\}$ 30: reorder SC31: 32: end while

After getting all the customer-vehicle assignments, a satellite-vehicle assignment algorithm is implemented. The only difference between the customer-vehicle assignment algorithm and the satellitevehicle assignment algorithm is that a satellite can be served by more than one vehicle. In this case, the serving cost approximation is made by product rather than by total demand and follows the same logic the customer-vehicle algorithm follows. The procedure can be found in Algorithm 2, where the initial values of parameters and sets are the values obtained at the end of Algorithm 1. At the end of Algorithm 2, each vehicle $k \in K$ has a set of nodes N_k to visit, as well as their respective demands characterized by the m_{pik} values. If $k \in V_D$, the x_{pfk} values from the MEMPHV formulation can be obtained from the m_{pik} values. Given that the routing costs in the objective function of MEMPHV depend only on the effective weight transported, instead of considering the detail by product, the total effective weight can be consolidated.

Algorithm 2 Satellite-vehicle assignment.

1: while $\max_{s \in S} \{\sum_{p \in \mathcal{P}} D'_{ps}\} \neq 0$ do if $(SC = \emptyset)$ then 2: 3: infeasible break 4: 5: else Select the first $SC_{skp} \in SC$ 6: if $(D'_{ps}\omega_p \leq \min\{\Omega'_{n(k)}, \Theta'_k\})$ and $(D'_{ps}\nu_p \leq \min\{\Lambda'_{n(k)}, \Phi'_k\})$ then $\Omega'_{n(k)} = \Omega'_{n(k)} - D'_{ps}\omega_p; \Lambda'_{n(k)} = \Lambda'_{n(k)} - D'_{ps}\nu_p$ $\Theta'_k = \Theta'_k - D'_{ps}\omega_p; \Phi'_k = \Phi'_k - D'_{ps}\nu_p$ 7:8: 9: $m_{psk} = m_{psk} + D'_{ps}$ 10: $D'_{ps} = 0$ 11: if $(k \notin K)$ then 12: $K = K \cup \{k\}$ 13:for $S_{s'kp} \in SC$ do 14: $S_{s'kp} = S_{s'kp} - \delta_k$ 15:end for 16:end if 17: $N_k = N_k \cup \{s\}$ 18:end if 19: 20: end if $SC = SC \setminus \bigcup_{k' \in \mathcal{K}} \{SC_{sk'p}\}$ 21:reorder SC22:23: end while

Thus, for each vehicle $k \in K$, we can formulate a flow problem as follows:

$$(SP^{(k)}) \quad \min \ \sum_{i \in N_k} \sum_{j \in N_k} r_{ij}(\rho_k w_{ij} + \gamma_k g_{ij})$$
s.t.
$$\sum_{i \in N_k} w_{ij} = 1 \qquad \forall j \in N_k$$

$$\sum_{i \in N_k} w_{ij} = \sum_{l \in N_k} w_{jl} \qquad \forall j \in N_k$$

$$g_{ij} \leq W'_k w_{ij} \qquad \forall i, j \in N_k$$

$$\sum_{i \in N_k} g_{ij} - \sum_{l \in N_k} g_{jl} = \bar{g}_j \qquad \forall j \in N_k \setminus \{n(k)\}$$

$$\sum_{i \in N_k} g_{in(k)} - \sum_{l \in N_k} g_{n(k)l} = -W'_{k0}$$

$$e_i - e_j - |N_k| w_{ij} \leq |N_k| - 1 \qquad \forall i, j \in N_k \setminus \{n(k)\}$$

$$e_f \leq e_i \qquad \forall f \in \mathcal{F} \cap N_k, \forall i \notin (\mathcal{F} \cup \{n(k)\}) \cap N_k$$

For $A_k = \{(i, j) : i, j \in N_k, i \neq j\}$ the decision variables are:

$$\begin{split} g_{ij} &: \text{effective weight passing throught } (i,j) \in A_k \\ w_{ij} &: \begin{cases} 1 & \text{if arc } (i,j) \in A_k \text{ is used} \\ 0 & \text{otherwise} \end{cases} \end{split}$$

 e_i : dummy variable for sequencing of node $i \in N_k \setminus \{n(k)\}$

and with auxiliary parameters:

- total effective weight for vehicle k: $W'_k = \sum_{p \in \mathcal{P}} \sum_{i \in N_k} m_{pik} \omega'_p$
- initial effective weight for vehicle k: $W'_{k0} = \sum_{p \in \mathcal{P}} \sum_{k' \in \mathcal{K} \setminus \{k\}} m_{pik} \omega'_p$

• effective weight demand at *i*:
$$\bar{g}_i = \begin{cases} -\sum_{p \in P_i} \sum_{j \in N_k, j \neq i} m_{pjk} \omega'_p & i \in \mathcal{F} \cap N_k \\ \sum_{p \in \mathcal{P}} m_{pik} \omega'_p & \text{otherwise} \end{cases}$$

Note. Decision variables for volume and weight are unnecessary since the objective function only depends on effective weight, and both volume and weight feasibility are checked in the previous steps.

5.2 Solution improvement

The initial solution can be improved by exchanging nodes between routes of different vehicles and removing vehicles or satellites when they do not have routes or nodes respectively assigned. The following functions are defined to simplify the algorithm notation:

- GetMostExpensiveNode (k, N_k, g_{ik}^k) : returns a node $j \notin \mathcal{F} \cup \{n(k)\}$ such that $\sum_{i \in N_k} r_{ij} \gamma_k g_{ij}^k$ is maximum, where g_{ij}^k are the optimal g_{ij} values from $SP^{(k)}$
- MinRoutingCost (k, N_k, m) : Computes the minimal cost route for vehicle k given a set of nodes N_k and the m values.

Algorithm 3 Initial solution improvement. 1: while iter \leq MaxIters do 2: for $k \in K$ do $i_k = \text{GetMostExpensiveNode}(k, N_k, m)$ 3: $RC_k = \text{MinRoutingCost}(N_k, m); RC'_k = \text{MinRoutingCost}(N_k \setminus \{i_k\}, m)$ 4: 5: $RC_{k^*} = \infty$ $RC'_{k^*} = \infty$ 6: 7: for $v \in K \setminus \{k\}$ do if $((N_k \cap N_v) \neq \emptyset)$ and $(\alpha_{i_k v} > 0)$ then 8: if $(\sum_{p \in \mathcal{P}} m_{pi_k k} \omega_p \leq \min\{\Omega'_{n(v)}, \Theta'_v\})$ and $(\sum_{p \in \mathcal{P}} m_{pi_k k} \nu_p \leq \min\{\Lambda'_{n(v)}, \Phi'_v\})$ then 9: if $(RC_{k^*} < \text{MinRoutingCost}(N_v \cup \{i_k\}, m))$ then 10: $RC_{k^*} = \text{MinRoutingCost}(N_v \cup \{i_k\}, m)$ 11: $k^* = v$ 12: $RC_{k^*} = \operatorname{MinRoutingCost}(N_v, m)$ 13:end if 14:end if 15:end if 16:17:end for if $(RC'_{k} + RC'_{k^{*}} > RC_{k} + RC_{k^{*}})$ then 18: $N_{k^*} = N_{k^*} \cup \{i_{k}^*\}$ $N_k = N_k \setminus \{i_k^*\};$ 19:for $p \in \mathcal{P}$ do 20: $m_{pi_kk^*} = m_{pi_kk}; \qquad m_{pi_kk} = 0$ 21: end for 22: else 23: $B = B \cup \{i_k\}$ 24:end if 25:26:for $k \in K$ do if $(\sum_{j \in N_k} w_{n(k)jk} < 1)$ then 27: $K = K \setminus \{k\}$ 28:end if 29:end for 30: for $s \in S$ do 31: if $(\sum_{j \in N_k} \sum_{k \in V_s} w_{sjk} < 1)$ then 32: $S = S \setminus \{s\}$ 33: 34: end if end for 35: iter = iter + 136: end for 37: 38: end while

5.3 Tests on random instances

The purpose of this section is to validate the proposed strategy and the rationality of the solutions provided by the model. For this, we have generated a set of 1000 realistic random instances, with $|\mathcal{F}| = 2$, $|\mathcal{D}| = 1$, $|\mathcal{S}| = 4$, $|\mathcal{C}| = 8$, $|\mathcal{P}_f| = 1 \quad \forall f \in \mathcal{F}$, and $|V_i| = 1 \quad \forall i \in \mathcal{N} \setminus \mathcal{C}$. Four different



Figure 2: Tours obtained for a small instance

"cities" were considered, as well as a LEZ depending on the city's radius. Most of the customers are placed on a LEZ. Satellites can only hold bikes, and delivering companies can have trucks or vans. Parameters Φ_k, Θ_k were randomly selected from Table 2. Regarding to the depots' capacities parameters, they were defined as follows: $\Lambda_i = \sum_{k \in V_i} \Phi_k, \Omega_i = \sum_{k \in V_i} \Theta_k$. Products' parameters were randomly selected from Table 1, and demands for every customer were randomly generated.

Fig. 2 represents the results obtained for a small instance. On the left size, the routing from the delivery node (2) is displayed and, on the right size, the routings from the satellites (3), (4), (5) and (6) are displayed.

Table 3 and Fig. 3 summarise the gap distribution. The exact solution is found for 40.6% instances.

Min. relative gap	0 %
Max. relative gap	102.86~%
Gap	pourcentage of instances
0	40.6%
<10%rel. gap	76.2%
<50%rel. gap	93.8%
>90% rel. gap	0.1%

Table 3:	Gap	distibution.
T (0)10 0	- Gup	anoundation



Figure 3: Distribution of relative gap between the exact solution and the heuristic solution.

The distribution of time variation between the exact method and the heuristic is provided in Table 4.



Figure 4: Distribution of time variation (%) between the exact method and the heuristic.

6 Conclusions

We have proposed and tested a practical mixed-integer linear programming model to optimise a multi-echelon multi-product distribution network characterised by a set of heterogeneous capacitated vehicles. The model is quite general, permitting to handle real-world instances in a unified way and reasonable computational cost. For the numerical resolution, we have applied a combination of linear optimisation and matheuristic strategies. The results on realistic instances provide insights and perspectives for further developments of practical computational tools for this very relevant problem in distributing products in large urban and suburban agglomerations. Our results can also motivate the building of new other efficient heuristics, as Particle Swarm or Ant Colony Optimization algorithms.

Our model allows determining an efficient distribution of various products from several firms to customers by interacting with a set of intermediate depots and satellites. The transport operator can use several types of vehicles for collecting or picking up from the different points of the network.

One of the major relevance of the problem we address here concerns the important contribution to the quality of life in medium and big cities, due to the impact in control the increasing level of congestion and the urban transport energy savings.

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