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Optimal Economic Envelope of Joint Open-Pit and Underground Mines*

Cecilia Julio
Department of Mining Engineering, University of Chile

Jorge Amaya
LP Laboratory, Centre for Mathematical Modelling, University of Chile

Amina Lamghari
COSMO Stochastic Planning Laboratory, McGill University

Nelson Morales*
Delphos Mine Planning Laboratory at the Advanced Mining Technology Center, & Department of Mining Engineering, University of Chile
Address: Avenida Tupper 2069, Santiago, Chile. Phone: +5698837980. Email: nelson.morales@amtc.cl

ABSTRACT

The costs and complexity of the mine operation increase as extraction in an open pit mine progresses, making the use of underground methods more and more appealing. A natural question is, therefore, what are the boundaries of the open pit and underground sections of the mine that maximize economic value for the company?

A few attempts have been made to answer this question, from sequential: open-pit first and then underground mining the remaining material, to complex simultaneous scheduling of both mines at different aggregation levels. The main focus of these works has been determining the optimal transition time.

In this work, we focus more on the definition of the economic envelope of the mines rather than time. Our motivation for this is the robustness of this decision. For this we present a model and algorithm to jointly determine the optimal envelopes of the open pit and underground sections of the mine.

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1 INTRODUCTION

The right selection of the extraction method becomes crucial at the moment to optimize the value of the project. In some cases, the deposit is mined completely by open – pit, as well as it can be mined completely by underground, but there are also deposits which geometry extent enable the development of a combined extraction method, where the decision is related to measure the "transition depth" (Bakhtavar et al. 2008) that will determines the beginning of the underground mining.

The transition from one methodology to another depends of the mining cost, which increase dramatically (Wang et al. 2012) in the way that the pit becomes deeper, as well as geological conditions (Chen et al. 2003) that affects the viability of the transition, ranging from stability of the pit slopes, minimum thickness of the crown pillar in between (Bakhtavar et al., 2008) to the rock characteristics that enable an underground extraction. The scope of keep mining is to continue achieving the production rate, so the best underground method would be block/panel caving (Hamrin, 2001).

This paper aim to develop an algorithm that calculates simultaneously both feasible envelops, for open – pit and underground extraction considering the minimum thickness of the crown pillar as an input from the data assumptions.

1.1 RELATED WORK

The transition problem became relevant since the current open pit project has started to consider the extraction by underground as means to extend the life of the mine considering the ore remained in the deposit. (Bakhtavar et al. (, 2008) developed a heuristic methodology, which started defining the ultimate pit limit based on block economic values and precedence constraints (Lerchs and Grossmann, 1965). Once this was settled, the boundary of the underground mine is defined by an analogous procedure, always considering the crown pillar between both methods. (Bakhtavar et al., (2008) also defined the thickness of the optimal crown pillar as a function of geomechanical parameters like span ratio and rock mass ratio, rock mass characteristics that should be considered to avoid a collapse (Carter et al. 1998)

Chen et al. (2013) pointed out the fact that there are two different deposits where the transition method could apply, ones that has horizontal extension, and the other ones that has vertical direction. Depending on the type, the problem will be subject to subsidence or to pit walls collapse.

Bakhtavar et al (2012) developed a mathematical model based on integer programming to compute both mining boundaries. Their work consider as parameters the size of the stope that guarantees stability, crown pillar constraints that reaches to get a minimum thickness and precedences constrains to ensure the slope angle in the open pit operation.

2 Problem Statement

2.1 General Description

The main interest is to study what is the optimal economic envelope when both methods, open pit and underground, are to be computed simultaneously. The problem considers the viability of panel/block caving methods as underground options in a way to keep the production rates of the open pit operation.

2.1.1 *Valuation of Blocks*

In order to proceed, each block is assigned two different economic values: the open pit and the underground. The value assigned as open pit corresponds to the best value between extracting and processing the block versus extracting the block and not processing it (waste). The value assigned as underground corresponds to the case of extracting and processing the cost. The main difference is that in the case of underground mining, extracting blocks has a higher cost because of the method and also the construction of an access to the level of extraction.

2.1.2 *Crown Pillar*

We consider the crown pillar as the minimum distance between the bottom of the pit and the top of the underground mine that ensures minimum safety requirements and stability conditions. According to Backtavar et al. (2010), the thickness of the crown pillar is

$$t = \frac{13.22 \cdot C^{0.33} \cdot S^{0.41} \cdot h^{0.56}}{\gamma_\gamma{}^{0.03} \cdot RMR^{0.66}}$$

where t is the thickness of the crown pillar, S is the stope span, h is the stope height, RMR is the Rock Mass Rating, C is the cohesive strength and γ_γ is the specific weight of rock mass. In our case, the parameters are presented in Table 1**¡Error! No se encuentra el origen de la referencia.**:

Parameter	C[Kg/ms^2]	S [m]	h [m]	RMR	γ_γ[Kg/m^{2}s^2]
Value	0.75	180	400	50	2.7

Table 1. Parameters for crown pillar thickness

2.1.3 *Dilution*

We used Laubscher graphical method (Laubscher 1994) to estimate the grade dilution in the underground mine. According to the author the point of dilution entry is

$$PDE(\%) = \frac{(H_c \cdot s - HIZ)}{H_c \cdot s} \cdot dcf \cdot 100$$

where H_c is the column height, s is the swallow factor, HIZ is the hight of interaction zone and dcf is the standard deviation factor. The parameters used in this paper are shown in Table 2.

Parameter	H_c [m]	HIZ [m]	s	dcf	RMR
Value	350	100	1.12	0.6	50

Table 2. Parameters to dilution calculation

2.2 Mathematical Modelling

2.2.1 *Notation*

$\boldsymbol{B} = \{1,2,\dots,N\}$ is the set of blocks. Each block $i \in \boldsymbol{B}$ has and open pit economic value $p(i)$ and underground value $q(i)$. $k(i)$ is the level of block i, that is an integer vertical coordinate. Levels go from 1 at the bottom of the block model to L on top.

Extraction through the pit is constrained by the slope of the pit walls expressed using *precedence arcs*. An arc is a pair $(i,j) \in \boldsymbol{B} \times \boldsymbol{B}$, representing the fact that block *j* has to be extracted before block i in order to gain access. We denote $\boldsymbol{P}$ the set of all pit precedences.

Extraction using underground is modelled similarly. For each block i there is a certain set of blocks that have to be extracted before it, but in this case these blocks are located below block i. We consider an arc set $\boldsymbol{Q} \subset \boldsymbol{B} \times \boldsymbol{B}$ encoding this.

Access to level k for the underground mine costs $R(k)$. Finally, the crown pillar consists of K levels (measured in blocks) and the maximum height of an underground extraction column is H, measured in block levels and not meters as H_C.

2.2.2 *Mathematical Formulation*

As mentioned before, we present a mathematical formulation of the problem to be solved.

The main variables of the problem correspond to the decision of extracting blocks using the open pit method or the underground one. We consider

$x_i^P = 1$ if block i is extracted by the open pit method and 0 otherwise

$x_i^U = 1$ if block i is extracted by the underground method and 0 otherwise

We also require variables to locate the crown pillar and the production level of the underground mine. For this, we consider variables

$y_k = 1$ if the bottom of the crown pillar is at level k or above

$z_k = 1$ if the bottom of the underground production level is at level k or above

¡Error! No se encuentra el origen de la referencia. summarizes the variable values and their interpretation.

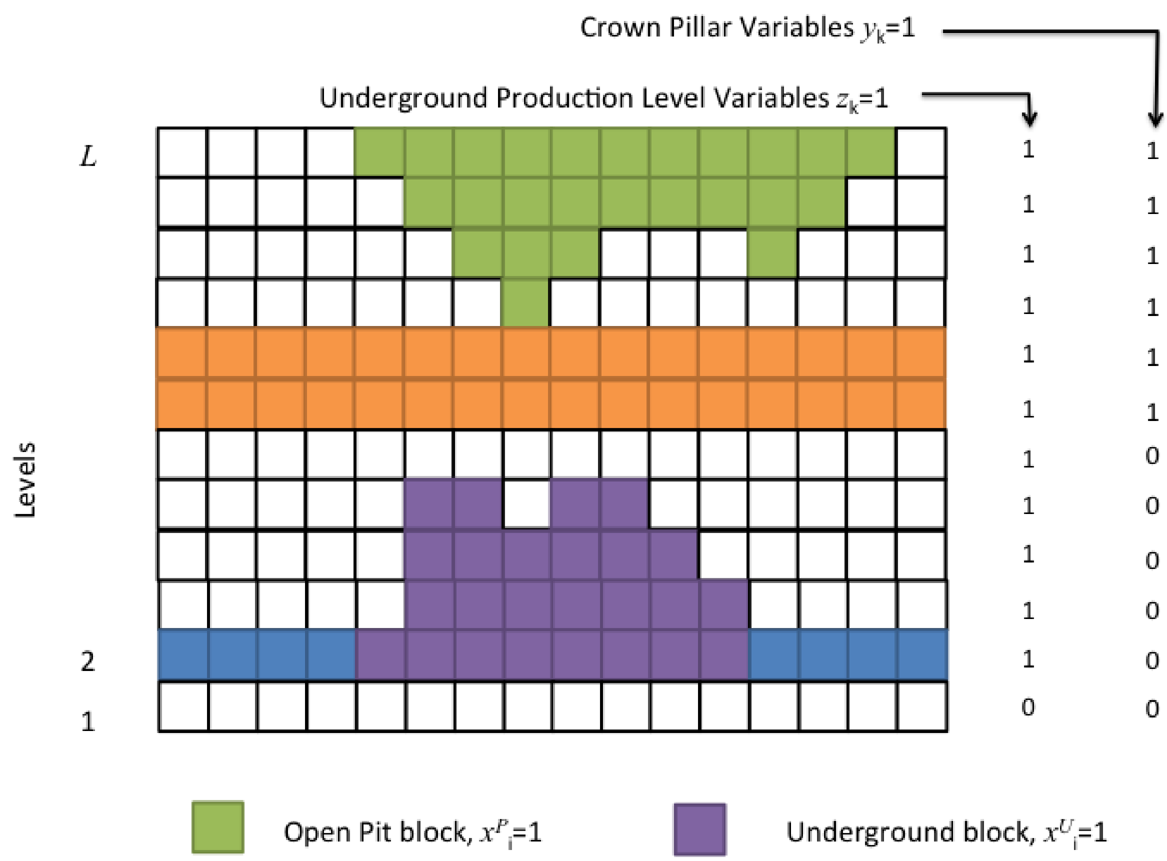

Figure 1. Problem variable definitions

Goal Function	$V = \sum_{i \in \boldsymbol{B}} p(i)\, x_i^P + q(i)\, x_i^U - R(1)\, y_1 - \sum_{k=2}^{L} R(k)(y_k - y_{k-1})$	
At most one crown pillar	$y_k \geq y_{k-1}$	$\forall k = 2, \ldots, L$
At most one underground production level	$z_k \geq z_{k-1}$	$\forall k = 2, \ldots, L$
Pit Precedences	$x_i^P \leq x_j^P$	$\forall (i,j) \in \boldsymbol{P}$
Underground Precedences	$x_i^P \leq x_j^P + z_k - z_{k-1}$	$\forall (i,j) \in \boldsymbol{Q}, k(i) > 1$
	$x_i^P \leq z_1$	$\forall (i,j) \in \boldsymbol{Q}, k(i) = 1$
Open pit only above the crown	$x_i^P \leq z_{k(i)-K}$	$\forall i, k(i) > K$

pillar		
No underground above Crown pillar	$x_i^U + y_k \leq 1$	$\forall k, \forall i, k\ \ i\ \ \geq k$
Maximum underground height	$x_i^U + z_1 \leq 1$ $x_i^U + z_k - z_{k-1} \leq 1$	$\forall\ k > 1, \forall i \in B, k\ \ i\ \ > k + H$

2.3 DESCRIPTION OF THE ALGORITHM

To solve the problem presented in the previous section we propose to parameterize the problem in terms of the crown pillar location and the production level location. When doing this, the optimal pit and underground mine can be computed independently by using simply ultimate pits computations which are fast and do not miss the optimal solution.

Let us call k_C the level of the bottom of the crown pillar and k_P the level of the production level of the underground mine. If $k_C = 0$, $k_P = 0$ corresponds to the pure open pit case and $k_C = 0, k_P \neq 0$ the pure underground case, the possible combinations of (k_C, k_P) are

$$X = \begin{array}{c} 0,0 \quad \cup \\ \{\ 0, k\ : k = 1, \dots, L - H\} \quad \cup \\ \{\ k_C, k_P\ : k_C = 1, \dots, L,\ \ k_P = 1, \dots k_C - H\} \end{array}$$

In order to describe the algorithm, we also denote $B\ \ k_1, k_1\ \ =\ \ i \in B\colon k_1 \leq k\ \ i\ \ \leq k_2$, the set of blocks with levels between k_1 and k_2.

ALGORITHM

1. Let v^* be the value of the ultimate pit computed on the whole block model, P^* such pit and $U^* = \emptyset$ (current underground mine)
2. For each $(k_C, k_P) \in X$:
    a. Compute ultimate pit in $B[k_C + K, L]$. Let v be its value.
    b. Compute best underground mine in $B[k_P, k_P + H]$. Let w be its value.
    c. If $v + w - R\ \ k_P\ \ > v^*$:
        i. Set $v^* = v + w - R(k_P)$
        ii. Set P^* to the pit obtained in 2.a.
        iii. Set U^* to the underground envelope from 2.b.
3. Return (P^*, U^*).

3 CASE STUDIES

In this section we briefly present two case studies against which we test the model and algorithm described in the previous section.

3.1 GOLD DEPOSIT

This block model has 82,932 blocks of size 10x10x10 [m^3] and mean gold grade of 0.104 [ppm]. The deposit covers an area of 570x780 [m^2] and a vertical extension of 570 [m] starting at the level -295. Some general information on the resource are presented in Figure 2.

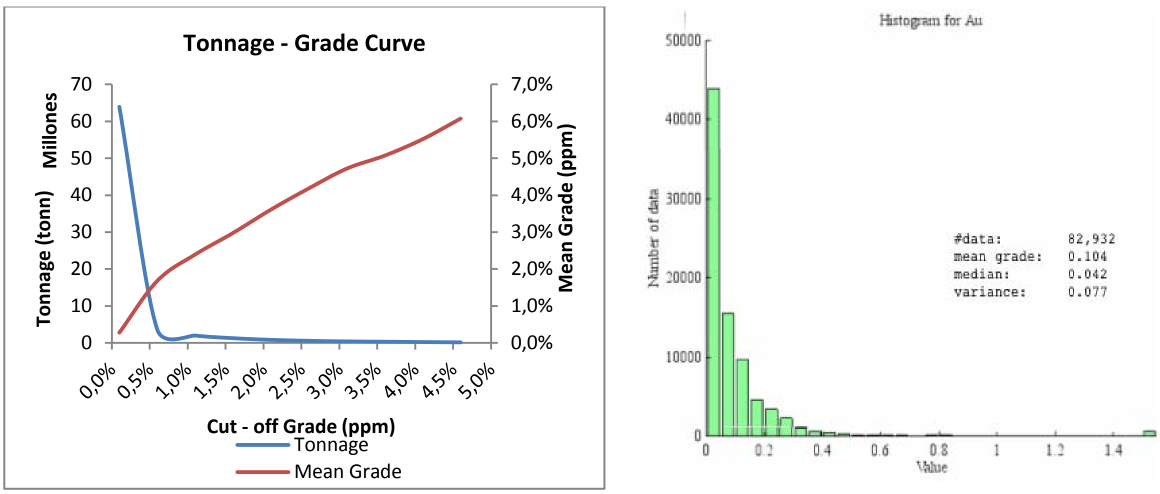

Figure 2. Tonnage-grade curves and grade distribution for Gold deposit case study

The parameters used to the optimization are shown in Table 3.

Open Pit & Underground	
Selling price	19.29 $(US\$/unit)$
Selling cost	N/A
Mining cost	1.8 $(US\$/ton)$
Proc cost – Oxide Ore	8.195 $(US\$/ton)$
Recovery	90%

Table 3. Cost and price parameters for the gold deposit

In the case of the underground extraction, considerations like the vertical mining rate and the maximum high of the extraction column are in Table 4:

Underground Parameters	
Max Column high	350 [m]
Area (grid 15x15 [m])	225 [m^2]
Vertical Mining rate	66 [m/year]
Development cost	3000 [US$/$m^2$]
Height of Interaction zone	100 [100]

Table 4. Parameters for the underground paremeterscaving mine

3.2 COPPER DEPOSIT

This deposit has 2,399,999 blocks of size 10x10x10 [m^3], with copper mean grade of 0.584 [%].The deposit covers an area of 1550x990 [m^2] and a vertical extension of 1490 [m] starting at the level 2755. General information on the resources is displayed in Figure 3.

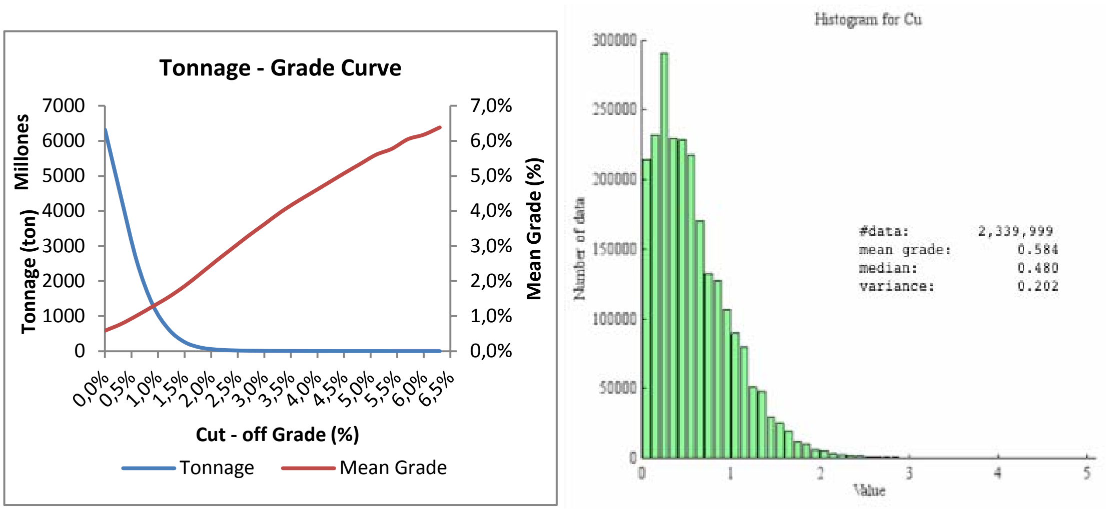

Figure 3. Tonnagegrade curve and histogram of the copper grade distribution for copper case study

Cost parameters and price are listed inTable 5.

Open Pit & Underground	
Selling price	3 $(US\$/lb)$
Selling cost	0.35 $(US\$/lb)$
Mining cost	1.8 $(US\$/ton)$
Proc cost – Oxide Ore	10 $(US\$/ton)$
Recovery	90%

Table 5. Cost and price parameters for the copper deposit.

Underground parameters are the same as in the gold deposit, except that the maximum column height is 350 in this case.

4 RESULTS AND ANALYSIS

4.1 GOLD DEPOSIT

In this case, the optimal pit limit reaches a value over MUS$350 while the underground extraction through block/panel caving is economically unfeasible. This is reasonable because gold is usually found disseminated in the ore body (like in veins), therefore its extraction is not well suited for massive underground caving methods.

The bottom of the optimal pit can be set at the level 75 of the deposit; however, under this level there is no massive underground method of extraction.

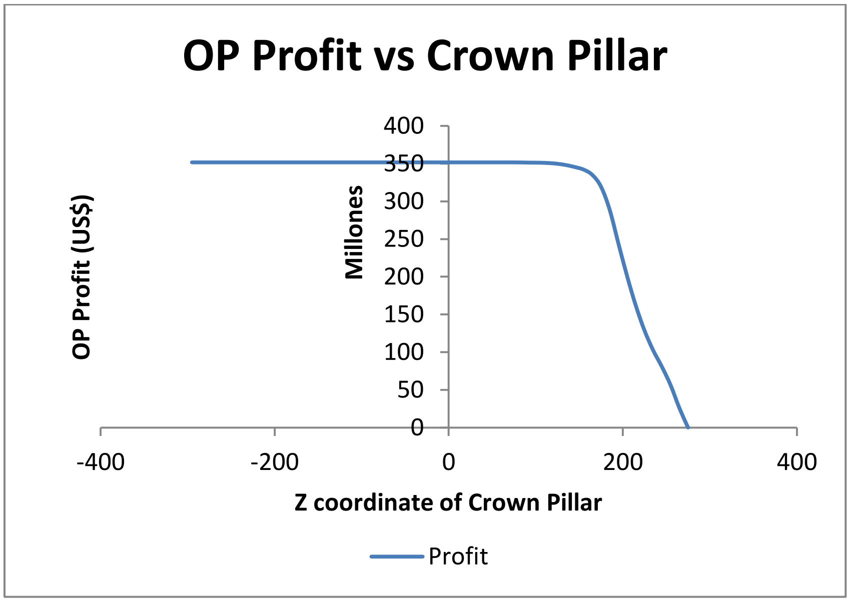

Figure 4. Profit curve of the gold deposit

4.2 Copper deposit

In the case of the copper deposit, the value decomposed by mining technology and the total value are presented in Figure 5 and Table 6. As it is expected, the value of the underground mine increases with the height of the crown pillar, as the development investment required to access the orebody decreases, but this increase does not pay off for the loss in value of having a smaller pit. Therefore, the optimum is to have a pit as deep as possible (level 3850) and then an underground mine at level 3400. It is also worth noting that, indeed, the best underground value (without considering development cost) is at level 3650.

CP top location [m]	Pit Value [MMUSD]	UG Foot Print Level [m]	UG Value w/o development [MMUSD]	UG Value [MMUSD]	Total Value [MMUSD]
3850	8,658	3400	4,838	3,993	12,651
3900	7,918	3450	4,959	4,164	12,083
3950	6,921	3500	5,213	4,468	11,389
4000	5,849	3550	5,359	4,664	10,512
4050	4,746	3600	5,524	4,879	9,624
4100	3,666	3650	5,570	4,975	8,642
4150	2,594	3700	5,649	5,104	7,699
4200	1,399	3750	5,637	5,142	6,542

Table 6. Underground, open pit and total envelope value

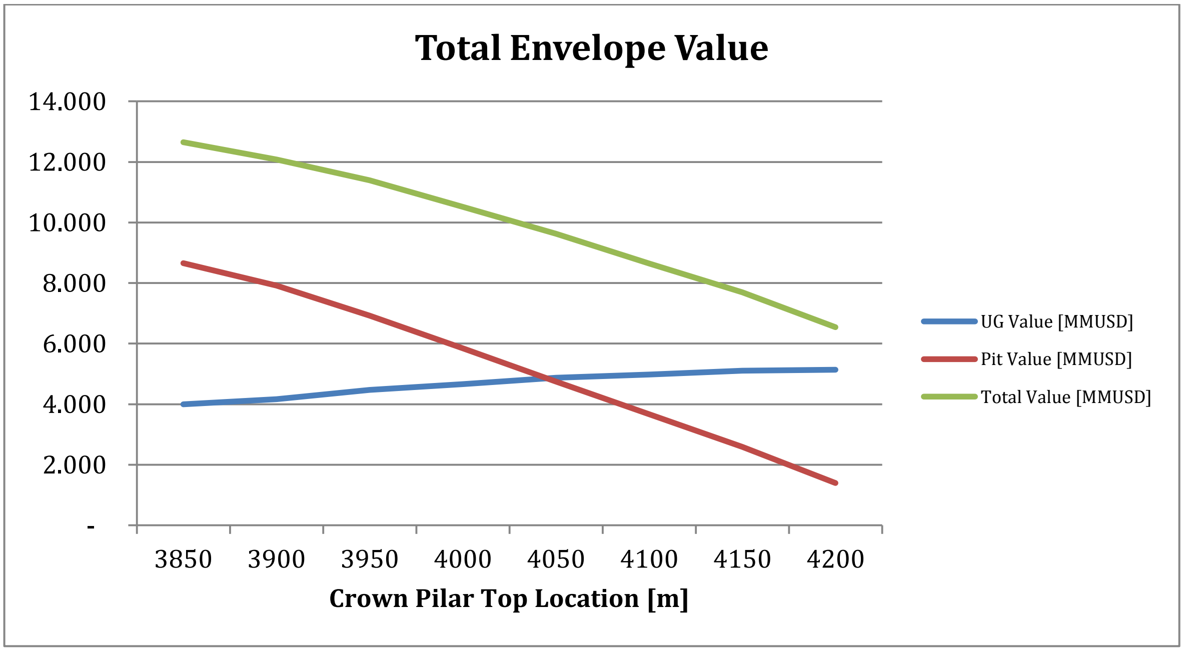

Figure 5. Envelope value depending on Crown pillar location

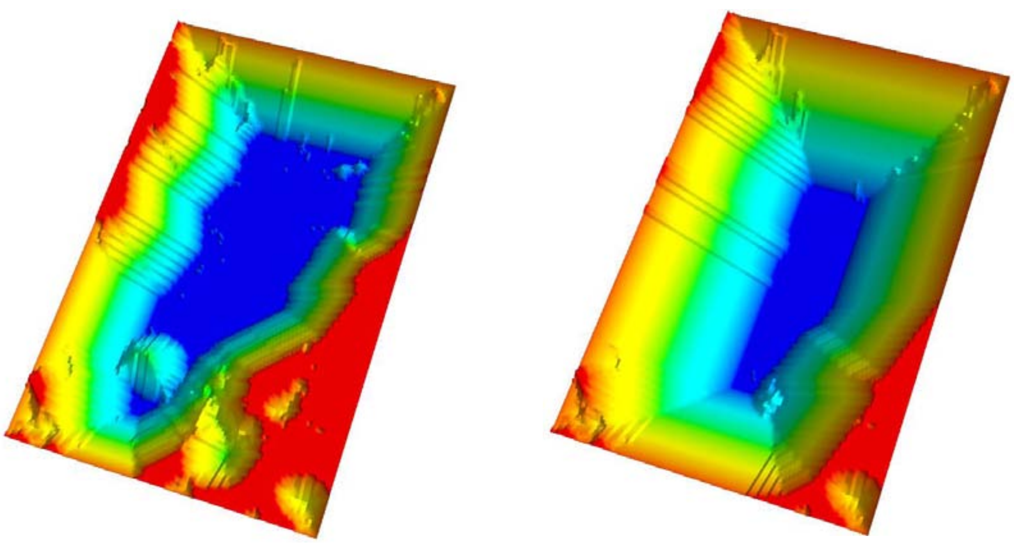

Figure 6 Final pit profiles. Left: Pit maximum depth is 4000[m]. Right: Pit maximum depth of 3850[m].

5 CONCLUSIONS

We have modelled the problem of selecting the best open pit and underground panel/block caving economic joint envelopes and implemented an algorithm to find the optimal solution of this problem.

We have tested the algorithm in two case studies, a gold deposit and a copper deposit. In the first case, as expected, the underground mine was not feasible for the underground method considered and therefore a more selective method should be tested. In the case of the copper mine, the algorithm finds the best solution.

The algorithm implemented is very fast, which encourages using it in more complex scenarios like studying the robustness of the joint envelope under geological uncertainty (that is grade variability, for example). Other potential research lines include scheduling of production and sensibility on prices.

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